PC 12 **SEC 8.1: UNDERSTANDING LOGARITHMS**

INVESTIGATE: LOGARITHMS

A. COMMON LOGARITHM BASE "10"

- Evaluate each logarithm to try to determine what this function does.

a) $\log 1 = 0$ (10^0 b) $\log 10 = 1$ (10^1 c) $\log 100 = 2$ (10^2 d) $\log 1000 = 3$ (10^3

e) $\log 0.1 = -1$ ($\frac{1}{10} = 10^{-1}$ f) $\log 0.01 = -2$ (10^{-2} g) $\log 0$ error ($10^{\square} \neq 0$) h) $\log(-1)$ error ($10^{\square} \neq -1$)

- **What do you think the LOG key is doing?**

- Use your calculator to determine each logarithm.
  Write each equation using an exponent with base 10

a) $\log 25 = \underline{1.39794}\ldots \rightarrow 10^{1.39794} = 25$

b) $\log 0.81 = \underline{-0.0915} \rightarrow 10^{-0.0915} = 0.81$

c) $\log(-2) = \underline{\text{error}} \rightarrow 10^{\square} = -2$ not possible

Do this on your calculator!
What do you notice?

- **What is another name for a logarithm?** exponents (inverses)

- **Why can't you take the logarithm of a negative number or zero?**
  because there is no exponent on base 10 which will give you an answer that is negative or zero

- **logarithm:** an exponent. In $x = c^y$, y is called the logarithm to the base c of x
- **common logarithm:** a logarithm with base 10

$\boldsymbol{\log_{10} x = y}$ **or** $\boldsymbol{\log x = y}$ **means** $10^y = x$

(base: 10; exponent: y)

notes: → logarithms are not restricted to a base of 10
→ any positive base except 1 can be used
→ Remember that a logarithm is just an exponent

B. LOGARITHM OF BASE "c"

- Write each logarithmic equation in exponential form using a base 2.

a) $\log_2 8 = 3$ b) $\log_2 32 = 5$ c) $\log_2(\frac{1}{4}) = -2$ d) $\log_2 2 = 1$ e) $\log_2 1 = 0$

$\log_2 2^3 = 3$ $2^3 = 8$ $2^5 = 32$ $2^{-2} = \frac{1}{4}$ $2^1 = 2$ $2^0 = 1$

- **logarithmic function:** a function of the form $y = \log_c x$, where $c > 0, c \neq 1, x > 0$ and is the inverse of the exponential function $y = c^x$

$\log_c x = y$ **means** $c^y = x$

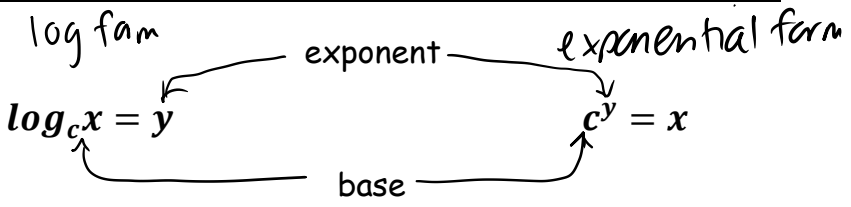
notes: → Why the restriction **c ≠ 1?** $1^y = x$ → it would always equal 1 (constant function)

→ Important properties:

$\log_c c = 1$ $\log_c 1 = 0$ $\log_c c^x = x$ $c^{\log_c x} = x$

$c^{\square} = c^x$

LOGARITHMS ARE EXPONENTS!!!



EVALUATING A LOGARITHM

EX. 1: Evaluate. Confirm you answer by writing the corresponding exponential equation.

a) $\log_6 36 = 2$ $6^{\square} = 36$

b) $\log_2 32 = 5$ $2^{\square} = 32$

c) $\log_b b^2 = 2$ $b^{\square} = b^2$

d) $\log_9 \sqrt[5]{81} = 2/5$ $9^{\square} = \sqrt[5]{81}$ $= \sqrt[5]{9^2}$ $= 9^{2/5}$

e) $\log_3 9\sqrt{3} = 5/2$ $3^{\square} = 9\sqrt{3}^1$ $= (3^2)(3^{1/2})$ $= (3^{4/2})(3^{1/2})$ $= 3^{5/2}$

f) $\log 0.0001 = -4$ $10^{\square} = 0.0001$ $= 10^{-4}$

EX. 2: Write each equation in exponential form.

a) $log_3 9 = 2$

$3^2 = 9$

b) $log_4 0.25 = -1$

$4^{-1} = 0.25$

c) $log_a b = c$

$a^c = b$

EX. 3: Write each equation in logarithmic form.

a) $3^5 = 243$

$\log_3 243 = 5$

b) $4^{-2} = \frac{1}{16}$

$\log_4 \frac{1}{16} = -2$

c) $p^q = r$

$\log_p r = q$

DETERMINE AN UNKNOWN IN AN EXPRESSION IN LOGARITHMIC FORM

EX. 4: Determine the value of x.

a) $log_4 x = -2$

$4^{-2} = x$

$\frac{1}{4^2} = x$

$x = \frac{1}{16}$

b) $log_{16} x = -\frac{1}{4}$

$16^{-1/4} = x$

$\frac{1}{16^{1/4}} = x$

$\frac{1}{\sqrt[4]{16}} = x$

$\frac{1}{2} = x$

c) $log_x 9 = \frac{2}{3}$

$x^{2/3} = 9$

$\left(\sqrt[3]{x^2}\right)^3 = (9)^3$

$\sqrt{x^2} = \sqrt{729}$

$x = 27$

$(x^{2/3})^3 = (9)^3$

$(x^{2 \cdot 1/2}) = (9^3)^{1/2}$

$x = 9^{3/2}$

$x = \left(\sqrt{9}\right)^3$

$x = 3^3$

$x = 27$

ESTIMATE THE VALUE OF A LOGARITHM

EX. 5: Without using graphing technology, estimate the value of $log_3 50$?

$\log_3 50 = x$

$3^x = 50$

guess & check

$3^3 = 27$

$3^4 = 81$

$3^{3.5} = 46.76$

$3^{3.56} = 49.95$

$3^{3.57} = 50.5$

$x = 3.561$

ESTIMATE THE VALUE OF A LOGARITHM

EX. 6: a) Write the inverse of ~~$f(x)$~~ $y = \left(\frac{1}{2}\right)^x$

Swap x + y + isolate y

$x = \left(\frac{1}{2}\right)^y \Longrightarrow \log_{1/2} x = y$

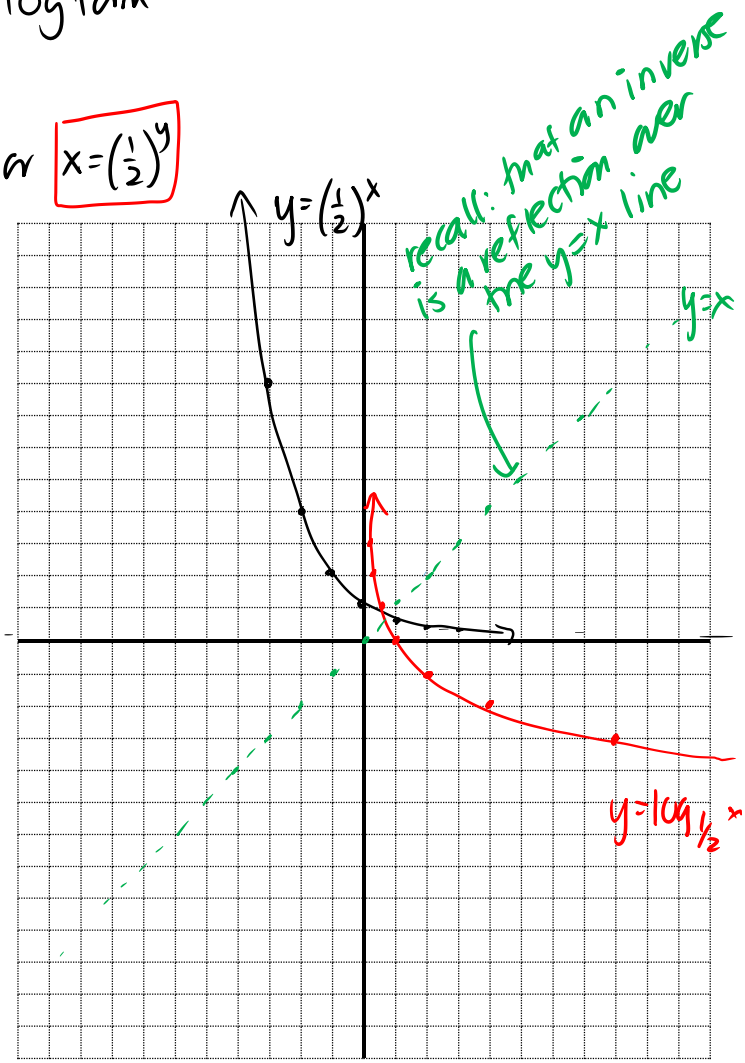
exp. form log form

b) Sketch the graphs of $f(x)$ and its inverse

$f(x) = \left(\frac{1}{2}\right)^x$ $f^{-1}(x) = \log_{1/2} x$ or $x = \left(\frac{1}{2}\right)^y$

x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8

x	y
8	-3
4	-2
2	-1
1	0
1/2	1
1/4	2
1/8	3



iii) Analyze each graph

$y = \frac{1}{2}^x$ $y = \log_{1/2} x$

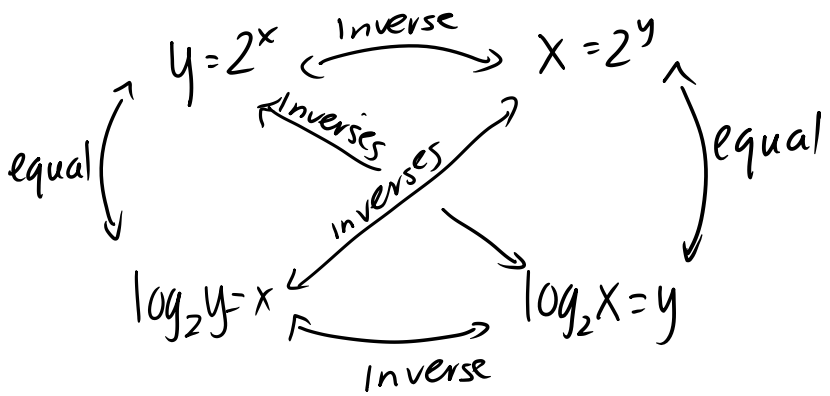
D: $x \in \mathbb{R}$ D: $x > 0$

R: $y > 0$ R: $y \in \mathbb{R}$

x-int: none x-int: 1

y-int: 1 y-int: none

asymptote: $y = 0$ asymptote: $x =$



AN APPLICATION OF LOGARITHMS

EX. 7: In 1935, American seismologist Charles R. Richter developed a scale formula for measuring the magnitude of earthquakes. The Richter magnitude, M, of an earthquake is defined as $\boldsymbol{M = log \frac{A}{A_0}}$

$\boldsymbol{A}$: amplitude of the ground motion (microns) – measured by a sensitive seismometer

$\boldsymbol{A_0}$: amplitude corrected for distance to actual earthquake, that would be expected for a "standard" earthquake

a) In 1946, an earthquake struck Vancouver Island. It had an amplitude that was $10^{7.3}$ times A_0. What was the earthquakes magnitude on the Richter scale?

M

$M = \log \frac{10^{7.3} A_0}{A_0} \Rightarrow 10^M = 10^{7.3} \quad \therefore 1 = 7.3$

$M = \log_{10} 10^{7.3}$

FYI: Vancouver Island's largest historic earthquake (and Canada's largest historic onshore earthquake) was a magnitude 7.3 event that occurred at 10:13 a.m. on Sunday June 23, 1946. The epicentre was in the Forbidden Plateau area of central Vancouver Island, just to the west of the communities of Courtenay and Campbell River.
http://www.earthquakescanada.nrcan.gc.ca/histor/20th-eme/1946/1946photos-eng.php

b) The strongest recorded earthquake in Canada struck Haida Gwaii, in 1949. It had a Richter reading of 8.1. How many times as great as A_0 was its amplitude?

$8.1 = \log \frac{A}{A_0} \Rightarrow (A_0) 10^{8.1} = \frac{A}{A_0}(A_0)$

log form

$A = 10^{8.1} A_0$

$A = 125892541\, A_0$

$\therefore$ 125,892,541 x as great as amplitude

c) Compare the seismic shaking of the 1949 Haida Gwaii earthquake with the 1946 Vancouver Island earthquake.

$\frac{10^{8.1}}{10^{7.3}} = 10^{0.8}$

$= 6.3$ x as great

ASSIGNMENT: **1) Worksheet 8.1: Calculating With Logs**
2) pg. 380 # 1-4, 6, 7, 9,11-13, 15, 19

PC 12 **WORKSHEET 8.1: CALCULATING WITH LOGS**

- Calculate each answer to 2 decimal places.
- Keep track of pairs of questions which have equal results
  - ➔ Label the first matching pair with A's (#4 & #5) since they have equal results… Label the next matching pair with B's … and so on…

Note: $log^2(x)$ must be entered as $log(x)^2$ on the calculator

1. $\log 739$
2. $7\log 39$
3. $\log(-6)$
4. $-\log 6$
5. $\log\left(\frac{1}{6}\right)$
6. $\log^2(5.2)$
7. $\log(5.2^2)$
8. $2\log 5.2$
9. $\frac{3}{2}\log(2.7^2)$
10. $3\log 2.7$
11. $-\frac{3}{2}\log^2(2.7)$
12. $10^{\log 2.7}$
13. $\log\left(10^{2.7}\right)$
14. $\log 2.7+5$
15. $\log(2.7+5)$
16. $\log 2.7+\log 5$
17. $\log(2.7\times 5)$
18. $\log\sqrt{\pi}$
19. $\frac{\log \pi}{2}$
20. $\log\frac{\pi}{2}$
21. $\log \pi-\log 2$
22. $\log 2\pi$
23. $2\log\pi$
24. $\log \pi^2$

ANSWERS:

1. 2.87
2. 11.14
3. undefined
4. –0.78
5. –0.78
6. 0.51
7. 1.43
8. 1.43
9. 1.29
10. 1.29
11. –0.28
12. 2.7
13. 2.7
14. 5.43
15. 0.89
16. 1.13
17. 1.13
18. 0.25
19. 0.25
20. 0.20
21. 0.20
22. 0.80
23. 0.99
24. 0.99

FYI: Vancouver Island's largest historic earthquake (and Canada's largest historic onshore earthquake) was a magnitude 7.3 event that occurred at 10:13 a.m. on Sunday June 23, 1946. The epicentre was in the Forbidden Plateau area of central Vancouver Island, just to the west of the communities of Courtenay and Campbell River.
http://www.earthquakescanada.nrcan.gc.ca/histor/20th-eme/1946/1946photos-eng.php

Earthquake map of Canada

Screen reader users, turn on the “expand abbreviations” configuration setting in your Screen Reader to hear entire words instead of abbreviations.