PC 12 **SEC 7.3: SOLVING EXPONENTIAL EQUATIONS FUNCTIONS**

REVIEW: EXPONENT LAWS (Mostly Review from Grade 9 & 10)

→ **a** and **b** are rational or variable bases AND **m** and **n** are rational exponents

	EXAMPLES
Product of Powers (multiplication) $(a^m)(a^n) = a^{m+n}$	$2^3 \cdot 2^4 = 2^7 = 128$
Quotient of Powers (division) $a \neq 0$ $(a^m)\div(a^n)$ or $\left(\frac{a^m}{a^n}\right) = a^{m-n}$	$\frac{2^6}{2} = 2^5$
Power of a Power $(a^m)^n = a^{m \cdot n}$	$(2^3)^4 = 2^{12}$
Power of a Product $(ab)^m = a^m b^m$	$(2x)^3 = 2^3x^3 = 8x^3$
Power of a Quotient $b \neq 0$ $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$	$\left(\frac{x}{y}\right)^5 = \frac{x^5}{y^5}$
Zero Exponent $a \neq 0$ $a^0 = 1$	$\left(\frac{2}{3}\right)^0 = 1$ $\sqrt{xxx} = x\sqrt{x}$
fractional **Rational Exponent** $a \neq 0,\ n \neq 0$ * flower power $(a^{\frac{1}{n}})^m = a^{\frac{m}{n}}$ or $\left(\sqrt[n]{a^m}\right) = \left(\sqrt[n]{a}\right)^m$	$(64^{\frac{1}{3}})^2 = 64^{2/3}$ $\sqrt[3]{64}^{\,2} = 16$ $(9x)^{\frac{3}{2}} = 9^{3/2}x^{3/2}$ $\sqrt[2]{9}^{\,3} \cdot \sqrt[2]{x^3} = 27x\sqrt{x}$
Negative Exponent $a \neq 0$ *reciprocals $a^{-n} = \left(\frac{1}{a^n}\right)$ or $a^n = \left(\frac{1}{a^{-n}}\right)$	$(x^{-3})^{\frac{1}{5}} = x^{-3/5} = \frac{1}{x^{3/5}} = \frac{1}{\sqrt[5]{x^3}}$ $25^{-1.5} = 25^{-3/2} = \frac{1}{25^{3/2}} = \frac{1}{\sqrt[2]{25^3}} = \frac{1}{125}$

INVESTIGATE : WRITING POWERS WITH THE SAME BASE

EXPONENTIAL FORM	2^7	2^6	2^5	2^4	2^3	2^2	2^1	2^0
STANDARD FORM	128	64	32	16	8	4	2	1

EXPONENTIAL FORM	2^{-7}	2^{-6}	2^{-5}	2^{-4}	2^{-3}	2^{-2}	2^{-1}	2^0
STANDARD FORM	$\frac{1}{128}$	$\frac{1}{64}$	$\frac{1}{32}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1

EXPONENTIAL FORM	3^4	3^3	3^2	3^1	3^0	3^{-1}	3^{-2}	3^{-3}	3^{-4}
STANDARD FORM	81	27	9	3	1	$\frac{1}{3}$	$\frac{1}{9}$	$\frac{1}{27}$	$\frac{1}{81}$

EXPONENTIAL FORM	4^4	4^3	4^2	4^1	4^0	4^{-1}	4^{-2}	4^{-3}	4^{-4}
STANDARD FORM	256	64	16	4	1	$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{64}$	$\frac{1}{256}$

EXPONENTIAL FORM	5^4	5^3	5^2	5^1	5^0	5^{-1}	5^{-2}	5^{-3}	5^{-4}
STANDARD FORM	625	125	25	5	1	$\frac{1}{5}$	$\frac{1}{25}$	$\frac{1}{125}$	$\frac{1}{625}$

EXPONENTIAL FORM	6^4	6^3	6^2	6^1	6^0	6^{-1}	6^{-2}	6^{-3}	6^{-4}
STANDARD FORM	1296	216	36	6	1	$\frac{1}{6}$	$\frac{1}{36}$	$\frac{1}{216}$	$\frac{1}{1296}$

INVESTIGATE: DIFFERENT WAYS TO EXPRESS EXPONENTIAL FUNCTIONS

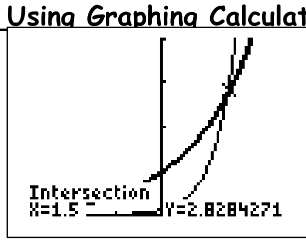
- **exponential equations**: an equation that has a variable as an exponent

A. Rewrite each side of the equation $2^x = 8^{x-1}$ with the same base if possible, then solve

Method 1: Using Graphing Calculator

$y_1 = 2^x$

$y_2 = 8^{(x-1)}$



$x = 1.5$

Method 2: Without Using Graphing Calculator

$2^x = (2^3)^{x-1}$

$2^x = 2^{3x-3}$ *drop base

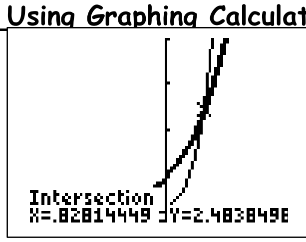
$\therefore$ $\underset{-x}{\cancel{x}} \underset{+3}{} = 3x \underset{-x}{} - \underset{+3}{\cancel{3}}$

$3 = 2x$

$\boxed{x = \frac{3}{2}}$

B. Rewrite each side of the equation $3^x = 4^{2x-1}$ with the same base if possible, then solve

Method 1: Using Graphing Calculator



Method 2: Without Using Graphing Calculator

trial & error

$3^x = (3^{\square})^{2x-1}$

$3^x = (3^{1.2618595})^{2x-1}$

$\therefore$ $x = 1.2618595(2x-1)$

$+1.26$ $\quad x \overset{-x}{} = 2.52x \overset{-x}{} - 1.26 \overset{+1.26}{}$

$\frac{1.26}{1.52} = \frac{1.52x}{1.52}$ $\quad x = \frac{1.26}{1.52} \cong 0.83$

CHANGE THE BASE OF POWERS

EX. 1: a) Write $27^{\frac{1}{3}}(\sqrt[3]{81})^{-2}$ as a power with base 3 b) Write $8^{\frac{2}{3}}(\sqrt{16})^{3}$ as a power with base 2

a)
$$(3^3)^{1/3}\cdot(3^4)^{2/3}$$
$$= 3^1\cdot 3^{8/3}$$
$$= 3^{1+8/3} \qquad 3^{3/3+8/3}$$
$$= 3^{11/3}$$

b)
$$(2^3)^{2/3}\cdot(2^4)^{3/2}$$
$$= 2^{6/3}\cdot 2^{12/2}$$
$$= 2^2\cdot 2^6$$
$$= 2^8$$

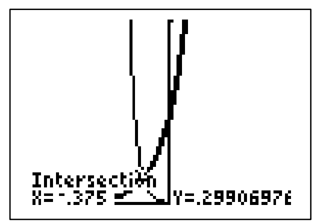
SOLVE AN EQUATION BY CHANGING THE BASE

EX. 2: a) Solve $25^x = (\frac{1}{125})^{2x+1}$

Method 1: Using Change of Base

$$(5^2)^x = (5^{-3})^{2x+1}$$
$$5^{2x} = 5^{-6x-3}$$
$$+6x \qquad +6x$$
$$\therefore\ 2x = -6x-3$$
$$8x = -3 \qquad x = -3/8$$

Method 2: Using Graphing Calculator

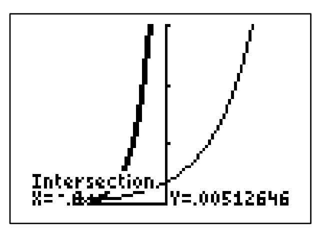


b) Solve $9^{4x} = 27^{x-1}$

Method 1: Using Change of Base

$$(3^2)^{4x} = (3^3)^{x-1}$$
$$3^{8x} = 3^{3x-3}$$
$$-3x \qquad -3x$$
$$\therefore\ 8x = 3x-3$$
$$5x = -3$$
$$x = -3/5$$

Method 2: Using Graphing Calculator



SOLVE AN EQUATION BY SYSTEMATIC TRIAL

EX. 3: Solve $2(5)^x = 3^{x+1}$

Method 1: Using Systematic Trial

try $x=1$ $\quad 2(5)^1 = 3^{1+1}$

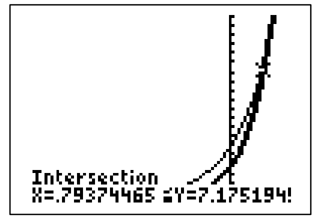
$10 = 9$

try $x=0.8$ $\quad 2(5)^{0.8} = 3^{0.8+1}$

$7.25 = 7.22$

etc. . .

Method 2: Using Graphing Calculator



SOLVE PROBLEMS INVOLVING EXPONENTIAL EQUATIONS WITH DIFFERENT BASES

EX. 4: Determine how long \$1000 needs to be invested in an account that earns 8.3% compounded semiannually before it increases to in value to \$1490.

$$A = P(1 + \frac{r}{n})^{nt}$$

$r = 0.083$ $\quad n = 2$

$P = 1000$

$A = 1490$

Method 1: Using Systematic Trial

$1490 = 1000\left(1 + \frac{0.083}{2}\right)^{2t}$

$1490 = 1000(1.0415)^{2t}$

$1.49 = (1.0415)^{2t}$

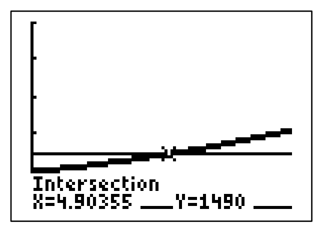
try $t=3$ $\quad 1.49 = (1.0415)^{2(3)}$

$1.49 = 1.27$

$t=5$ $\quad 1.49 = (1.0415)^{10}$

$1.49 = 1.50$

Method 2: Using Graphing Calculator



$t=4.9$ $\quad 1.49 = (1.0415)^{2(4.9)}$

$1.49 = 1.49$ ✓

$\therefore$ 4.9 years.

ASSIGNMENT:	**1) Worksheet 7.3: Solving Exponential Equations**
	2) pg. 364 # 1-7, 10, 12, 13, *18

PC 12 **WORKSHEET 7.3: SOLVING EXPONENTIAL EQUATIONS**

1. Solve $2^{4x-1} = 8^{x}$

Check:

2. Solve $6^{x+1} = 36^{x-1}$

Check:

3. Solve $2^{x} = 5$, using

Method 1: Sytematic Trial

Method 2: Graphing Calculator

Check:

Method 3: Logarithms