PC 12 **SEC 7.2: TRANSFORMATIONS OF EXPONENTIAL FUNCTIONS**

INVESTIGATE TRANSFORMING AN EXPONENTIAL FUNCTION

$$f(x) = a(c)^{b(x-h)} + k \quad \text{OR} \quad f(x) = a(B)^{b(x-c)} + d$$

A. The Effects of Parameter k on the function $f(x) = a(c)^{b(x-h)} + k$

- Graph the set of functions on the grid. vertical translations

i) $f(x) = 2^x$ ii) $f(x) = 2^x + 3$ iii) $f(x) = 2^x - 4$

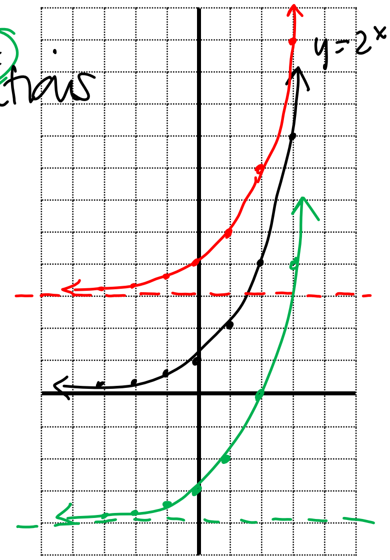
x	y
-3	1/8
-2	¼
-1	½
0	1
1	2
2	4
3	8

x	y
-3	3⅛
-2	3¼
-1	3½
0	4
1	5
2	7
3	11

up 3 units

x	y
-3	-3⅞
-2	-3¾
-1	-3½
0	-3
1	-2
2	0
3	4

down 4



B. The Effects of Parameter h on the function $f(x) = a(c)^{b(x-h)} + k$

- Graph the set of functions on the grid. horizontal translations

i) $f(x) = 2^x$ ii) $f(x) = 2^{x+3}$ iii) $f(x) = 2^{x-4}$

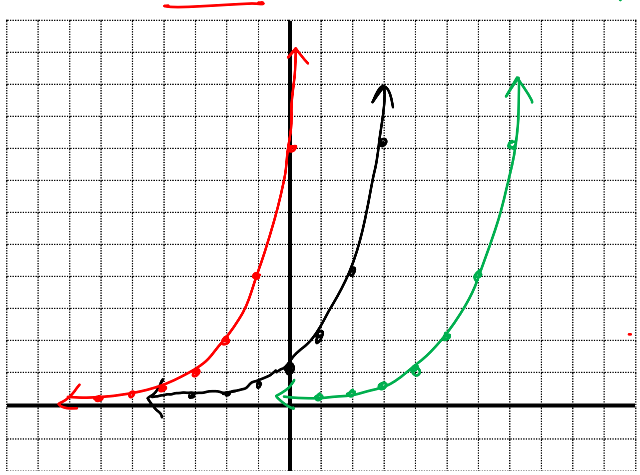
x	y
-3	1/8
-2	¼
-1	½
0	1
1	2
2	4
3	8

x	y
-6	1/8
-5	¼
-4	½
-3	1
-2	2
-1	4
0	8

left 3

x	y
1	1/8
2	¼
3	½
4	1
5	2
6	4
7	8

right 4



C. The Effects of Parameter a on the function $f(x) = a(c)^{b(x-h)} + k$

- Graph the set of functions on the grid. vert. exp or comp or reflection

i) $f(x) = 2^x$

x	y
-3	1/8
-2	¼
-1	½
0	1
1	2
2	4
3	8

ii) $f(x) = 3(2)^x$

x	y
-3	3/8
-2	3/4
-1	3/2
0	3
1	6
2	12
3	24

iii) $f(x) = \frac{1}{3}(2)^x$

x	y
-3	1/24
-2	1/12
-1	1/6
0	1/3
1	2/3
2	4/3
3	8/3

iv) $f(x) = -(2)^x$

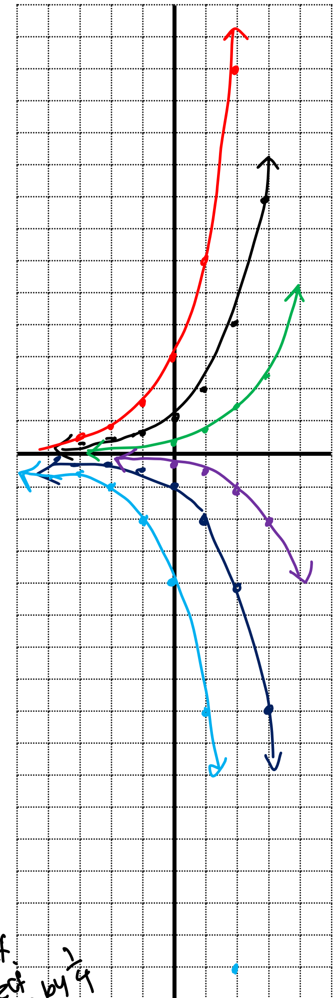
x	y
-3	-1/8
-2	-¼
-1	-½
0	-1
1	-2
2	-4
3	-8

v) $f(x) = \boxed{-4}(2)^x$

x	y
-3	-1/2
-2	-1
-1	-2
0	-4
1	-8
2	-16
3	-32

vi) $f(x) = -\frac{1}{4}(2)^x$

x	y
-3	-1/32
-2	-1/16
-1	-1/8
0	-1/4
1	-1/2
2	-1
3	-2



D. The Effects of Parameter b on the function $f(x) = a(c)^{b(x-h)} + k$

- Graph the set of functions on the grid. horz exp or comp or reflect.

i) $f(x) = 2^x$

x	y
-3	1/8
-2	¼
-1	½
0	1
1	2
2	4
3	8

ii) $f(x) = \boxed{(2)^{3x}}$ horz comp by 1/3

x	y
-1	1/8
-2/3	¼
-1/3	½
0	1
1/3	2
2/3	4
1	8

iii) $f(x) = (2)^{\frac{1}{3}x}$ horz exp 3

x	y
-9	1/8
-6	¼
-3	½
0	1
3	2
6	4
9	8

iv) $f(x) = (2)^{-x}$ horz reflect.

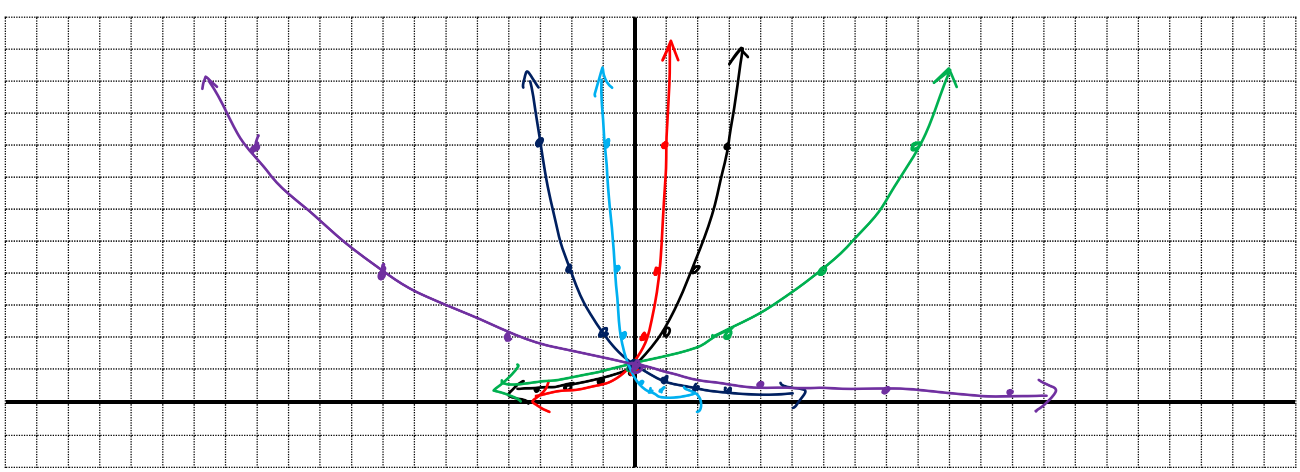
x	y
3	1/8
2	¼
1	½
0	1
-1	2
-2	4
-3	8

v) $f(x) = (2)^{-4x}$ reflect comp by 1/4

x	y
3/4	1/8
1/2	¼
1/4	½
0	1
-1/4	2
-1/2	4
-3/4	8

vi) $f(x) = (2)^{-\frac{1}{4}x}$

x	y
12	1/8
8	¼
4	½
0	1
-4	2
-8	4
-12	8



SUMMARY: Graphing Multiple Transformations

$y = a(c)^{b(x-h)} + k$ OR $y - k = a(c)^{b(x-h)}$

$y = af(b(x-c)) + d$

1) Identify the **"base graph"** and key points
2) Identify all changes (order matters):
   a) **Stretches** (vertical or horizontal) & **Reflections** (over x-axis or y-axis)
   b) **Translations** (vertical or horizontal)
3) Apply each changes to "base graph" coordinates to create newly transformed graph.

- **parameters & variables for $y = a(c)^{b(x-h)} + k$**
  Parameters a, b, h, and k are constants that define the relationship between the variables x and y in the exponential function

APPLY TRANSFORMATIONS TO SKETCH A GRAPH

Careful: Graphing Form?

EX. 1: Consider the base function $\boxed{y = 3^x}$ For the transformed function

a) $\boxed{y = 2(3)^{-x-4}}$ $= 2(3)^{-(x+4)}$

i) State parameters, describe transformations and show mapping notation.

Parameter	Transformation	Mapping notation
a = 2	vert exp by 2	$(x, y) \rightarrow$
b = -1	reflect horz (over y-axis)	$(x, y) \rightarrow$
h = -4	left 4	$(x, y) \rightarrow$
k = 0	no vert.. shift	$(x, y) \rightarrow$

ii) Create a table of values and graph the transformation

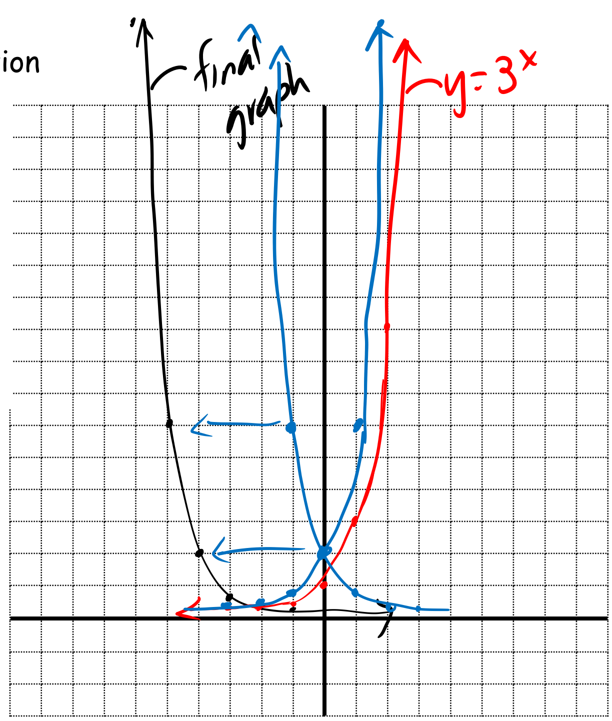
$y = 3^x$

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

$-x-4$

$2y$

x	y
-1	2/27
-2	2/9
-3	2/3
-4	2
-5	6
-6	18
-7	54



iii) Analyze the transformed graph

D: $x \in \mathbb{R}$ R: $y > 0$

x-int: none y-int: $2/81$

let $x = 0$ $y = 2(3)^{-0-4}$

$= 2(3)^{-4} = 2\left(\frac{1}{3}\right)^4 = 2\left(\frac{1}{81}\right) = \frac{2}{81}$

asymptote: $y=0$

EX. 1 cont'd Consider the base function $\boxed{y = 3^x}$ For the transformed function

b) $y = -\frac{1}{2}(3)^{\frac{1}{4}(x)} + 5$

j) State parameters, describe transformations and show mapping notation.

Parameter	Transformation	Mapping notation
a = $-\frac{1}{2}$	vert comp by $\frac{1}{2}$ + vert reflect (x-axis)	$(x,y) \rightarrow$
b = $\frac{1}{4}$	horz exp by 4 —	$(x,y) \rightarrow$
h = 0	no horz shift	$(x,y) \rightarrow$
k = 5	up 5	$(x,y) \rightarrow$

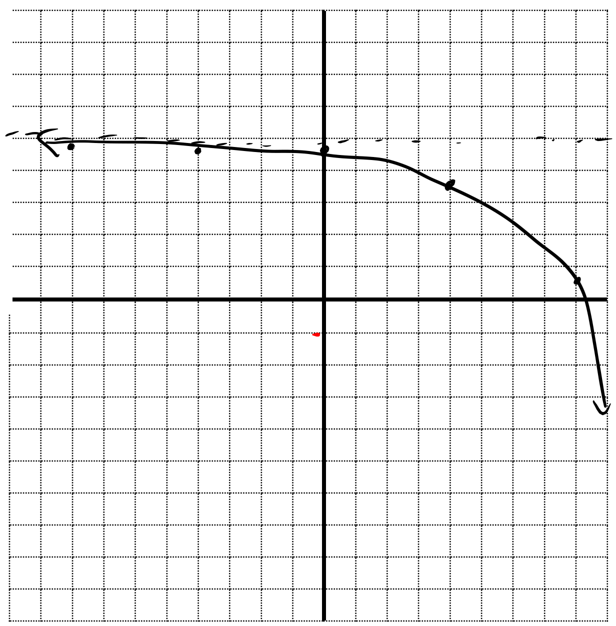
ii) Create a table of values and graph the transformation

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27

$4x$

$-\frac{y}{2}+5$

x	y
-12	4.98
-8	4.94
-4	4.83
0	4.5
4	3.5
8	0.5
12	-8.5



iii) Analyze the transformed graph

D: $x \in \mathbb{R}$ R: $y<5$

x-int: 8.38 y-int: 4.5

find on graphing calc.

asymptote: $y=5$

USE TRANSFORMATIONS OF AN EXPONENTIAL FUNCTION TO MODEL A SITUATION

EX. 2: Describe how the parameters in the transformed functions relates to info given.

a) \$1000 invested at 12% compounded quarterly for t years can be modeled by the equation $A = 1000(1 + \frac{0.12}{4})^{4t}$ = $1000(1.03)^{4t}$

Parameter	Transformation
a = 1000	Starting amount (principal)
b = 4	goes in every 1/4 of a year
h = 0	
k = 0	

EX. 2 cont'd

b) A radioactive sample of radium (Ra-225) has a half-life of 15 days. The mass, M, (grams) remaining over time, t, (days) can be modeled by the function $M = 60\left(\frac{1}{2}\right)^{\frac{t}{15}}$ $y = 0.5^x$

Parameter	Transformation
a = 60	← starting/initial value
b = 1/15	← half life time (= 15)
h = 0	
k = 0	

EX. 3: A cup of water is heated to $100°C$ and then allowed to cool in a room with an air temperature of $20°C$. The temperature, T, in degrees Celsius, is measured every minute as a function of time, m, in minutes, and these points are plotted on a coordinate grid. It is found that the temperature of the water decreases exponentially at a rate of 25% every 5 min. A smooth curve is drawn through the points, as shown on the graph.

a) What is the transformed exponential function in the form $\boxed{y = a(c)^{b(x-h)} + k}$ that can represent the situation?

$T = a(1-0.25)^{m/5} + 20$

1 pt (0

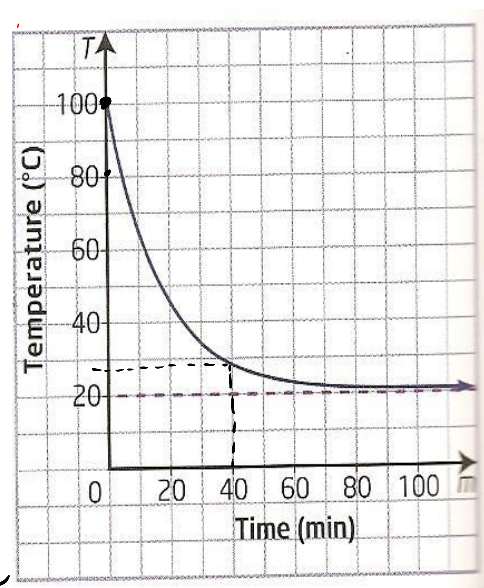
pick a pt (0, 100)

$100 = a(0.75)^{0/5} + 20$

$100 = a + 20$

$a = 80$

$T = 80(0.75)^{t/5} + 20$



Compare function with graph

Graph: $m = 40$ predict $T \approx 28$

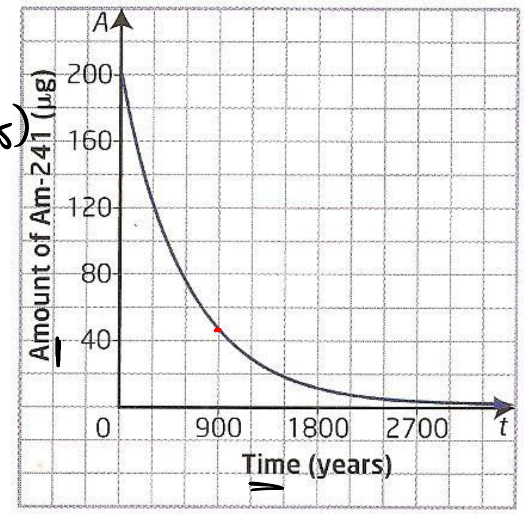
Function: $m = 40$ $T = 80(0.75)^{40/5} + 20 = 28°C$

b) Describe how each of the parameters in the transformed function relates to the information provided.

Parameter	Transformation
a = 80	difference btw initial temp + air temp
b = 1/5	interval of time (5 min)
h = 0	
k = 20	room temp

EX. 4: The radioactive element americium (Am) is used in household smoke detectors. Am-241 has a half-life of approximately 432 years. The average smoke detector contains 200 μg of Am-241.

a) What is the transformed exponential function that models the graph showing the radioactive decay of 200 μg of Am-241.



time elapsed (in yrs)

$$A = 200\left(\frac{1}{2}\right)^{t/432}$$

432 ← half-life time (in yrs)

final amount

initial amount

decay time (half life)

b) Describe how the parameters in the transformed functions relates to info given.

Parameter	Transformation
a = 200	vert exp by 200 → initial amount
b = 1/432	horz exp by 432 → interval of half-life
h = 0	
k = 0	

ASSIGNMENT: 1) Worksheet 7.2 Transforming Exponential Functions
2) pg. 354 # 1-3, 5, 6, 7, 13, [redacted]

PC 12 **WORKSHEET 7.2: TRANSFORMING EXPONENTIAL FUNCTIONS**

A. <u>MODELLING GROWTH</u>

1) In 1996, Kelowna's population was approximately 89 000 and was growing at a rate of 3.33% per year. Write an equation that models this population.

Parameter	Transformation
a	
b	
h	
k	

2) A swarm of insects can multiply tenfold in 30 days. The equation that models this swarms population, P, over time, t, given the initial population of C bees is $\boldsymbol{P = C(10)^{\frac{t}{30}}}$

Parameter	Transformation
a	
b	
h	
k	

3) The population, P million, of British Columbia can be modeled by the equation $\boldsymbol{P = 2.76(1.022)^{n}}$, where n is the number of years since 1981.

Parameter	Transformation
a	
b	
h	
k	

B. MODELLING DECAY

4) Radioactive phosphorus (P-32) is a common isotope used in DNA studies. It has a half-life of 14.3 days. The equation which shows the percent, P, of this radioactive substance after t days is $\boldsymbol{P = 100(\frac{1}{2})^{\frac{t}{14.3}}}$

Parameter	Transformation
a	
b	
h	
k	

5) The percent, P, of caffeine left in your body can be modelled as a function of the elapsed time, t hours, by the equation $\boldsymbol{P = 100(0.87)^t}$

Parameter	Transformation
a	
b	
h	
k	

6. For every meter, n, that you descend into water, 5% of light is blocked. The formula for the amount when decreasing to create an exponential function that gives the percent, A, of light remaining for n meters is $\boldsymbol{A = 100(1 - 0.05)^n}$

Parameter	Transformation
a	
b	
h	
k	