PC 12 **SEC 7.1 CHARACTERISTICS OF EXPONENTIAL FUNCTIONS**

INVESTIGATE EXPONENTIAL FUNCTION

- **exponential function:** a function of the form $y = c^x$, where c is a constant ($c > 0$) and x is a variable

A. Growth: $y = c^x$, $c > 1$,

ex. $\boxed{y = 2^x}$

D: $x \in \mathbb{R}$

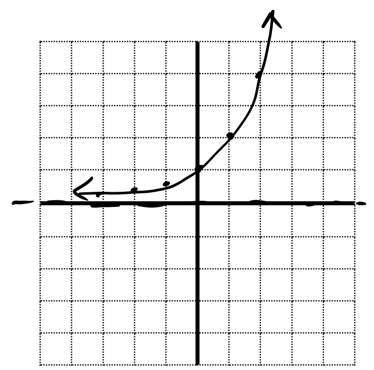
R: $y > 0$

x-int: none

y-int: 1

x	y
-3	1/8
-2	1/4
-1	1/2
0	1
1	2
2	4
3	8

$y = 2^{-3} = \frac{1}{2^3} = \frac{1}{8}$



The function is increasing

asymptote: y=0 (x-axis) → horizontal asymptote

applications: population growth, money/investments

B. Decay: $y = c^x$, $0 < c < 1$,

ex. $y = \left(\frac{1}{2}\right)^x$

D: $x \in \mathbb{R}$

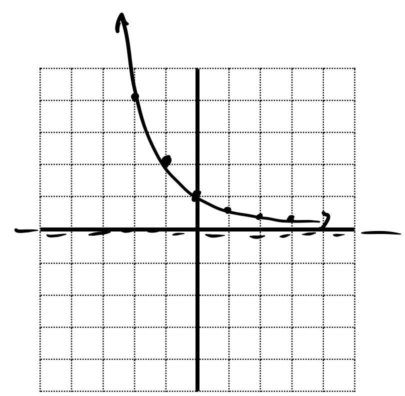
R: $y > 0$

x-int: none

y-int: 1

x	y
-3	8
-2	4
-1	2
0	1
1	1/2
2	1/4
3	1/8

$y = (\frac{1}{2})^{-3} = 2^3 = 8$



The function is decreasing

asymptote: y=0

applications: radioactive decay, population decline, drugs leave your system

ANALYSE THE GRAPH OF AN EXPONENTIAL FUNCTION

EX. 1: Sketch the graph of each function.

a) $y = 3^x$ **Method A:** **Using a Table of Values** **Method B:** **Using a Graphing Calculator**

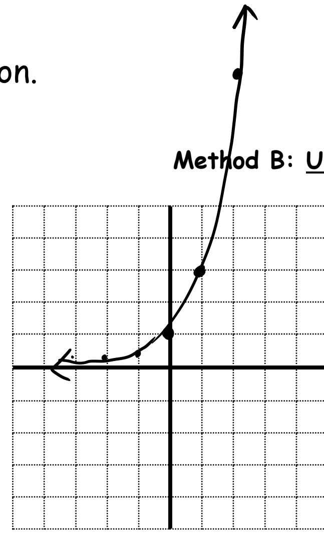
D: $x \in \mathbb{R}$

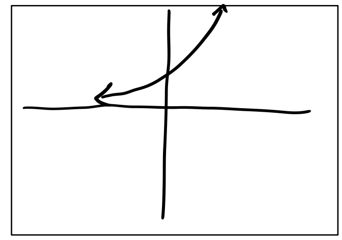
R: $y > 0$

x-int: none

y-int: 1

x	y
-3	1/27
-2	1/9
-1	1/3
0	1
1	3
2	9
3	27





The function is increasing

asymptote: $y=0$

b) $y = (\frac{1}{3})^x$ **Method A:** **Using a Table of Values** **Method B:** **Using a Graphing Calculator**

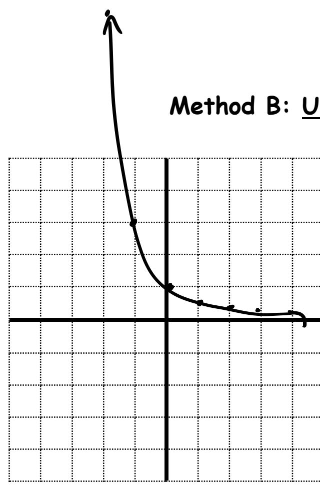
D: $x \in \mathbb{R}$

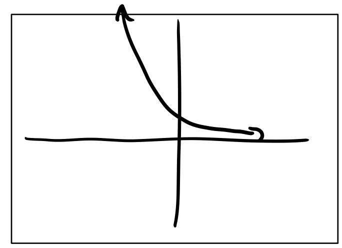
R: $y > 0$

x-int: none

y-int: 1

x	y
-3	27
-2	9
-1	3
0	1
1	1/3
2	1/9
3	1/27





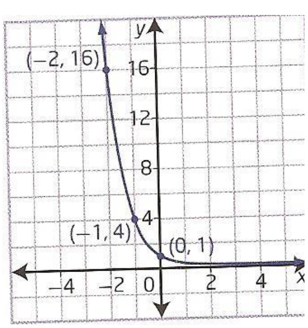
The function is decreasing

asymptote: $y=0$

WRITE THE EXPONENTIAL FUNCTION GIVEN ITS GRAPH

EX. 2: What function of the form $y = c^x$ can be used to describe the graph shown?

a)



decay function (decreasing)

x	y
-2	16
-1	4
0	1
1	1/4
2	1/16
3	1/64

$r = \frac{t_2}{t_1} = \frac{4}{16} = \frac{1}{4}$

$y = \left(\frac{1}{4}\right)^x$

Verify your answer

Check $(-1, 4)$

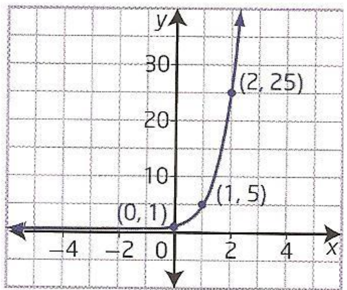
$4 = \left(\frac{1}{4}\right)^{-1}$

$4 = 4^1$

$4 = 4$ ✓

EX. 2 continued What function of the form $y = c^x$ can be used to describe the graph shown?

b)



growth function (increasing)

x	y
-2	1/25
-1	1/5
0	1
1	5
2	25

$r = \frac{t_2}{t_1} = \frac{25}{5} = 5$

$y = 5^x$

Verify your answer

check w/ (2,25)

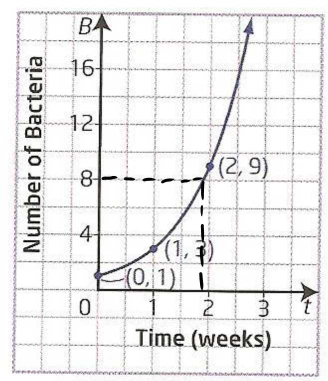
$25 = 5^2$

$25 = 25$ ✓

APPLICATION OF AN EXPONENTIAL FUNCTION

- **exponential growth:** an increasing pattern of values that can be modeled by a function of the form $y = c^x$, where $c > 1$
- **exponential decay:** a decreasing pattern of values that can be modeled by a function of the form $y = c^x$, where $0 < c < 1$

EX. 3: A certain bacteria population triples every week. This is modeled by the exponential graph shown.



a) What is the initial population? 1

What is the population approaching as time increases? no limit

b) What is the domain: $x \geq 0$ and range: $y \geq 1$

c) Write the exponential growth model that relates the number of bacteria (B) to the time (t) in weeks.

t	B
0	1
1	3
2	9

$B = 3^t$

d) Estimate how many days it would take for the bacteria to increase to 8 times the quantity on day 1.

from the graph

~ 1.8 weeks

algebraically

$8 = 3^t$

guess + check

$3^{1.8} = 7.22$

$3^{1.9} = 8.06$

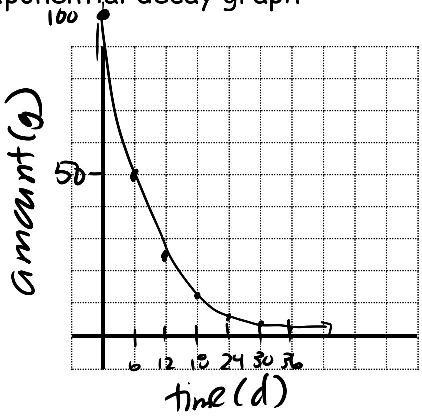
~1.9 weeks × 7

= ~13.3 days

- **half-life:** the length of time for an unstable element to spontaneously decay to one half its original mass (can be modeled with an exponential decay graph

ex. 100 grams of a substance with a half-life of 6 days

Time (day)	*Amount (grams)*
0	100
6	50
12	25
18	12.5
24	6.25
30	3.125
36	1.5625

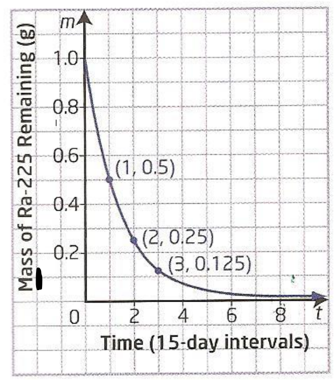


EX. 4: A radioactive sample of radium (Ra-225) has a half-life of 15 days. The mass (grams) remaining over time (days) can be modeled by the exponential graph shown.

a) What is the initial mass of Ra-225 in the sample? 1 g

What is the mass of Ra-225 approaching as time increases? 0

b) What is the domain: $x \geq 0$ and range: $0 < m \leq 1$

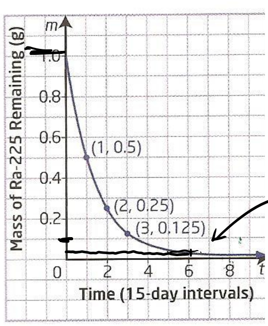


c) Write the exponential decay model that relates mass to time (15 day intervals)

$m = \left(\frac{1}{2}\right)^t$

d) Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.

Method A: Using Graph



$1.0g \times \frac{1}{30} = 0.033g$

$t = 6$

6×15 days $= 90$ days

Method B: Using Table

$r = 0.5$

x	y
1	0.5
2	0.25
3	0.125
4	0.0625
5	0.03125
6	

$\sim 4.9 \times 15$ days $= 73.5$ day

Method C: Using model

$M = \left(\frac{1}{2}\right)^t$

guess + check

$\boxed{\left(\frac{1}{2}\right)^{4.9} = 0.0335}$

$\left(\frac{1}{2}\right)^{4.8} = 0.0359$

$\left(\frac{1}{2}\right)^{5} = 0.03125$

$\sim 4.9 \times 15 = 73.5$ days

ASSIGNMENT: 1) Worksheet 7.1 Characteristics of Exponential Graphs

2) pg. 342 # 1-7, 10, 12, *15

PC 12 **WORKSHEET 7.1: Characteristics of Exponential Graphs**

n	compounding
1	Annually
2	Semi-annually
4	Quarterly
6	Bi-monthly
12	monthly
52	weekly
365	Daily

A. MODELLING GROWTH → INCREASING BY r% $A = P\,(1 + r)^t$

COMPOUND INTEREST:

$$A = P(1 + \frac{r}{n})^{nt}$$

A: accumulated (final) amount
P: principal (initial) amount
r: rate of interest
n: compounding period
t: time in years

1. Suppose you want to make a lot of money and do very little to earn it. One way to do this is to start a long-term investment. You are going to put $1000 in the bank at 11% compounded annually for t years.

a) Use the compound interest formula then write a simplified exponential function

$$A = 1000(1 + \frac{0.11}{1})^{1t} \quad = \quad \rule{6cm}{0.4pt}$$

b) Use the graphing calculator to sketch the function.

Choose appropriate window dimensions!

domain: 0-25 years on x-axis
range: 0 – 10 000 dollars on y-axis

c) What is the amount of the investment after 8 years?

2nd $\boxed{\text{TRACE}}$ 1:

d) How long did it take the investment to double?

1) Plug in: Y_2 = 2000 (double)
2) 2nd $\boxed{\text{TRACE}}$ 5: INTERSECT

e) If the original investment were $5500, rewrite the equation and determine how much money you would have in 8 years.

2. Grandma gives you $1500 and you do some great investing and get an average rate of return of 15.5% over a 10 year period. Write the exponential equation and determine the value of your 10 year investment.

$$A = P(1 + \frac{r}{n})^{nt}$$

3. The population, P million, of British Columbia can be modeled by the equation $P = 2.76(1.022)^n$, where n is the number of years since 1981.

2.2%

initial popl$^{\underline{n}}$

2.76 million is the population in 1981

a) Use the graphing calculator to sketch the function.

Choose appropriate window dimensions!

domain: ~~1970 – 2040~~ 0 – 56 year on x-axis

range: 0 – 10 millions on y-axis

b) Determine the population projected for 2021?

c) Estimate when the population might reach 5 million.

starting value 100% → none of light at top has been blocked

B. MODELLING DECAY → DECREASING BY r% $A = P\,(1-r)^t$

4. For every meter, n, that you descend into water, 5% of light is blocked. Use the formula for the amount when decreasing to create an exponential function that gives the percent, A, of light remaining for n meters.

   a) Write a function to represent this situation. $A = 100(1-0.05)^n$

   b) Use the graphing calculator to sketch the function.

   Choose appropriate window dimensions!

   domain: 0-50 meters on x-axis
   range: 0 – 100 percent on y-axis

   c) What percent of light remains at 6.5 meters down?

   d) At what depth below the surface is 50% of the light still remaining?

   e) If 7% of the light were blocked for every meter you descend, rewrite the equation and determine how much light would remain at 5 meters below the surface.

5. Coffee, tea, cola, and chocolate contain caffeine. When you consume caffeine, the percent, P, left in your body can be modelled as a function of the elapsed time, t hours, by the equation $P = 100(0.87)^t$

   a) Use the graphing calculator to sketch the function.

   Choose appropriate window dimensions!

   domain: 0-24 hours on x-axis
   range: 0 – 100 percent on y-axis

   b) Determine the percent of caffeine in your body after 5 hours.

   c) Estimate how many hours it will take until only 30% of the caffeine remains.