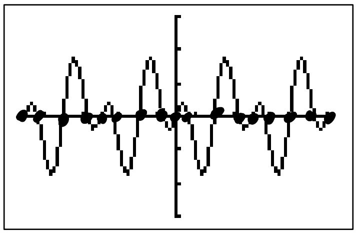
PC 12 **SEC. 6.4 SOLVING TRIGONOMETRIC EQUATIONS USING IDENTITIES**

INVESTIGATE SOLVING TRIGONOMETRIC EQUATIONS

- To solve some trigonometric equations, you need to make substitutions using trigonometric identities. This involves expressing the equation in terms of one trigonometric equation.

1. Solve $y = \boxed{sin2x} - sinx$ over the domain $-720° \leq x \leq 720°$.
   Make a sketch of the graph and describe it in words.



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2. Use the double angle identity to rewrite the equation $y = sin2x - sinx$ in terms of single trigonometric functions

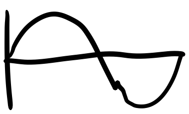
$y = 2\sin x\cos x - \sin x$

$y = (\sin x)(2\cos x - 1)$

3. Solve $y = sin2x - sinx$ over the domain $\boxed{-720° \leq x \leq 720°}$.
   Use the equivalent equation from (2) above to solve it algebraically.

2, $\frac{\pi}{6}$, $\sqrt{3}$, $\frac{\pi}{3}$, 1

S A T C



$0 = \sin x(2\cos x - 1)$

$x = 0, 180, 360°\ldots \leftarrow \sin x = 0$ or $2\cos x - 1 = 0$

$\cos x = \frac{1}{2} \rightarrow x = 60°, 300°$

$\boxed{\begin{aligned} x &= 180°n \\ &= 60° + 360°n \\ &= 300° + 360°n \end{aligned}} \; n \in I$

REVIEW: SOLVE TRIGONOMETRIC EQUATIONS (NOTES 4.4 and 5.4)

Use processes learned in previous grades to solve equations

➔ isolate variables, square roots, factoring (difference of squares, trinomial factoring including decomposition, grouping two and two), quadratic formula, long or synthetic division etc.

Use processes learned in 4.3 notes (p.21-22 EX.4) to find angles given trigonometric ratios

① Ignore sign; Use your calculator or special triangle to find reference angle, θ_r (or points on the unit circle for possible quadrantal angles)

② Use sign of ratio, "ASTC" and θ_r to sketch all possible angles in standard position

③ State the measure(s) of the possible angles in the required domain (use coterminal angles when necessary add/subtract full rotations as needed)

Use replacement method when the period is compressed or expanded in 5.4 notes (p.26 EX.2-3)

① Use replacement → let $\theta = "bx"$

② Solve for θ (reference angle (θ_r), quadrants (ASTC), find θ_1 *and* θ_2)

③ Replace each θ with "bx", then solve each equation for x. → $bx_1 = \theta_1$ and $bx_2 = \theta_2$

④ Find the general solution → add multiples the period (p) to each solution (x) → $x \pm pn$, $n \in I$

SOLVE BY SUBSTITUTING TRIGONOMETRIC IDENTITIES & FACTORING

- Don't forget to identify any non-permissible values when solving.

EX. 1: Solve each equation algebraically over the domain $0 \le x \le 2\pi$

a) $sin2x - \cos x = 0$

$2\sin x\cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0$ $\qquad$ $2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

$\theta_R = \pi/6$

$$\boxed{\begin{matrix} x_1 = \pi/2 & x_3 = \pi/6 \\ x_2 = 3\pi/2 & x_4 = 5\pi/6 \end{matrix}}$$

b) $2cosx + 1 - sin^2x = 3$

$2\cos x + \cos^2 x - 3 = 0$

$\cos^2 x + 2\cos x - 3 = 0$

let $a = \cos x$

$a^2 + 2a - 3 = 0$

$(a-1)(a+3) = 0$

$a - 1 = 0$ $\qquad$ $a + 3 = 0$

$a = 1$ $\qquad$ $a = -3$

$\cos x = 1$ $\qquad$ $\cos x = -3$ not possible

$\boxed{x_1 = 0 \quad x_2 = 2\pi}$

SOLVE AN EQUATION WITH A QUOTIENT IDENTITY SUBSTITUTION

EX. 2: a) Solve the equation $sin^2x = \frac{1}{2}tanx\ cosx$ algebraically over the domain $0° \le x \le 360°$

$\sin^2 x = \frac{1}{2}\frac{\sin x}{\cancel{\cos x}}\cancel{\cos x}$

$\sin^2 x = \frac{1}{2}\sin x$

$2\sin^2 x = \sin x$

$2\sin^2 x - \sin x = 0$

$\sin x(2\sin x - 1) = 0$

$\sin x = 0$ or $2\sin x - 1 = 0$

$\sin x = \frac{1}{2}$

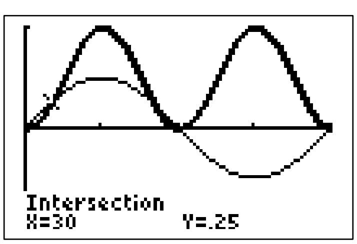
$x_1 = 0$ $\qquad$ $x_4 = 30°$ $\qquad$ $\theta_R = 30°$

$x_2 = 180°$ $\qquad$ $x_5 = 150°$ $\qquad$ S A T C

$x_3 = 360°$

b) Verify your answer graphically. Solve the equation

Non-permissible values?



$y_1 = \sin(x)^2$

$y_2 = \frac{1}{2}\tan x\cos x$

$\cos x \ne 0$

$\therefore x \ne 90°, 270°$

DETERMINE THE GENERAL SOLUTION FOR A TRIGONOMETRIC EQUATION

EX. 3: Algebraically solve $cos2x = cosx$. Give $\boxed{\text{general solutions}}$ expressed in radians.

all perm.

Non-permissible values?

$$2\cos^2 x - 1 = \cos x$$
$$2\cos^2 x - \cos x - 1 = 0$$

let $a = \cos x$

$-2 + 1 = -1$

$-2 \times 1 = -2$

$$2a^2 - a - 1 = 0$$
$$2a^2 - 2a + a - 1 = 0$$
$$2a(a-1) + 1(a-1) = 0$$
$$(2a+1)(a-1) = 0$$
$$2a + 1 = 0 \qquad a - 1 = 0$$
$$a = -\tfrac{1}{2} \qquad a = 1$$

$\cos x = -\frac{1}{2}$ OR $\cos x = 1$

$\theta_R = \frac{\pi}{3}$

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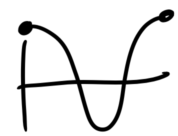


$x_1 = \frac{2\pi}{3} + 2\pi n, n \in I$

$x_2 = \frac{4\pi}{3} + 2\pi n, n \in I$

$x_3 = 2\pi n, n \in I$

DETERMINE THE GENERAL SOLUTION USING RECIPROCAL IDENTITIES

EX. 4: Algebraically solve $3cosx + 2 = 5secx$. Give general solutions expressed in degrees.

Non-permissible values?

$\cos x \neq 0$

$\therefore x \neq 90°, 270°$

$$(\cos x)(3\cos x + 2) = 5\left(\frac{1}{\cancel{\cos x}}\right)(\cancel{\cos x})$$
$$3\cos^2 x + 2\cos x = 5$$
$$3\cos^2 x + 2\cos x - 5 = 0$$

let $a = \cos x$

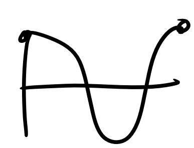
$5 + -3 = 2$

$5 \times -3 = -15$

$$3a^2 + 2a - 5 = 0$$
$$3a^2 - 3a + 5a - 5 = 0$$
$$3a(a-1) + 5(a-1) = 0$$
$$(3a+5)(a-1) = 0$$
$$3a + 5 = 0 \text{ or } a - 1 = 0$$
$$a = -\tfrac{5}{3} \qquad a = 1$$

$\cos x = -\frac{5}{3}$ or $\cos x = 1$

not possible



$$\boxed{x = 360°n, n \in I}$$

ASSIGNMENT: 1) **Worksheet 6.4**
2) **pg. 320 # 1-6, 8, 9, 11, 14, 16 *, 18**

PC 12	**WORKSHEET 6.4: SOLVING TRIGONOMETRIC EQUATIONS**

- **Solve each equation algebraically. Answers are provided for you to check.**
- **You may need to use identities to rewrite the equation before solving.**

1. Solve each equation for θ, with $0 \leq \theta \leq 360°$

a) $cos\theta + 1 = 0$ $\boldsymbol{\theta = 180°}$

b) $tan\theta(csc\theta + 2) = 0$ $\boldsymbol{\theta = 0°, 180°, 210°, 330°, 360°}$

c) $sec^2\theta + 2sec\theta = 0$ $\boldsymbol{\theta = 120°, 240°}$

d) $sin2\theta - cos\theta = 0$ $\boldsymbol{\theta = 30°, 90°, 150°, 270°}$

e) $2cos^2\theta + 3sin\theta - 3 = 0$ $\boldsymbol{\theta = 30°, 90°, 150°}$

f) $4cos^2\theta - 3 = 0$ $\boldsymbol{\theta = 30°, 150°, 210°, 330°}$

2. Solve each equation (or inequality) for θ, with $0 \leq \theta \leq 2\pi$

a) $3sec\theta - cos\theta - 2 = 0$ $\boldsymbol{\theta = 0, 2\pi}$

b) $2cos^4\theta - 3cos^2\theta + 1 = 0$ $\boldsymbol{\theta = 0, \frac{\pi}{4}, \frac{3\pi}{4}, \pi, \frac{5\pi}{4}, \frac{7\pi}{4}, 2\pi}$

c) $3tan^2\theta - 1 = 0$ $\boldsymbol{\theta = \frac{\pi}{6}, \frac{5\pi}{6} \frac{7\pi}{6}, \frac{11\pi}{6}}$

d) $sin\theta \leq 0 - 1 = 0$ $\boldsymbol{\pi \leq \theta \leq 2\pi}$

3. Solve each equation for θ, with $0 \leq \theta \leq 360°$

a) $2cos^2\theta - cos\theta = 1$ $\boldsymbol{\theta = 0°, 120°, 240°, 360°}$

b) $tan^2\theta - 3 = 0$ $\boldsymbol{\theta = 60°, 120°, 240°, 300°}$

c) $sin\theta + 2sin\theta cos\theta = 0$ $\boldsymbol{\theta = 0°, 120°, 180°, 240°, 360°}$

d) $cos2\theta + cos\theta = 0$ $\boldsymbol{\theta = 60°, 180°, 300°}$

4. Solve each equation for θ, with $0 \leq \theta \leq 2\pi$

a) $2sinxcosx + \sqrt{3}cosx = 0$ $\boldsymbol{\theta = \frac{\pi}{2}, \frac{4\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}}$

b) $cot^2\theta + 1 = 0$ $\boldsymbol{\theta = no\ solution\ (explain?)}$

c) $sin^2\theta + sin\theta cos\theta = 0$ $\boldsymbol{\theta = 0, \frac{3\pi}{4}, \pi, \frac{7\pi}{4}, 2\pi}$

d) $2 + sec\theta = 0$ $\boldsymbol{\theta = \frac{2\pi}{3}, \frac{4\pi}{3}}$