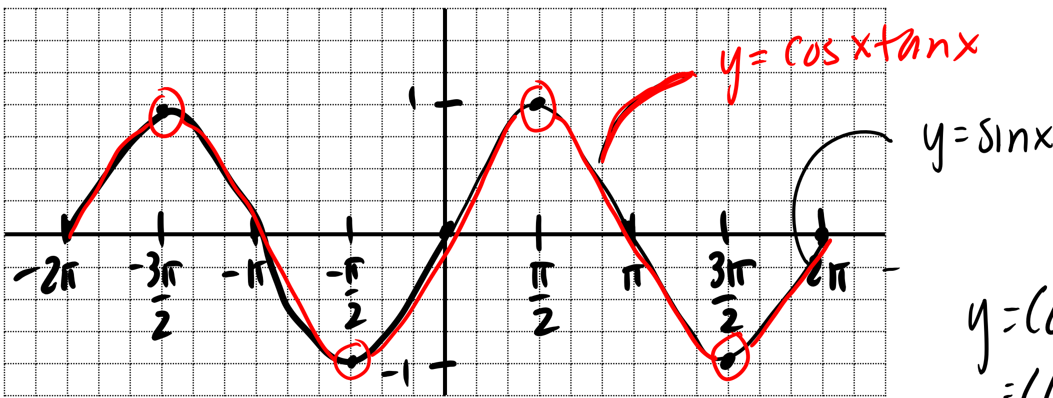
PC 12 **SEC 6.1: RECIPROCAL, QUOTIENT & PYTHAGOREAN IDENTITIES**

INVESTIGATE COMPARING TWO TRIGONOMETRIC EQUATIONS

1. Graph the curves $y = sinx$ and $y = cosx\ tanx$ over the domain $-360° \le x \le 360°$.



$y = \cos x \tan x$
$= \cos(90)\tan(90)$
$= (0)(\text{undef}) = \text{undef}$

2. Complete the table of values for each

x	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = sinx$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0

x	0	30°	60°	90°	120°	150°	180°	210°	240°	270°	300°	330°	360°
$y = cosx\ tanx$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	error	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	err	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	0

- What do you notice?

Same graphs except that the $y = \cos x \tan x$ graph has holes (undefined values)

3. Use your knowledge $tanx = \frac{sinx}{cosx}$ to simplify the expression $y = cosx\ tanx$

$y = \cos x \tan x$

$y = \cancel{\cos x}\left(\frac{\sin x}{\boxed{\cancel{\cos x}}}\right)$ but $\cos x \neq 0$

$y = \sin x$

4. a) Are the curves $y = sinx$ and $y = cosx\ tanx$ identical? Explain.

no, because y=cosx tan x has some holes (undefined values)

b) Why was it important to look at the graphs and the table of values?

because you might miss the non-permissible values if you don't use the table

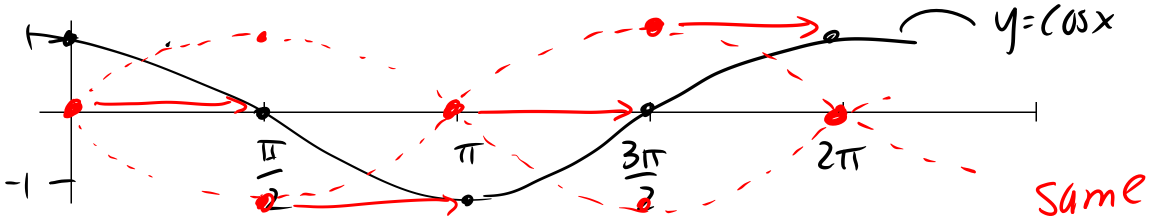
5. What are the non-permissible values of x in the equation $sinx = cosx\ tan\ x$?

$x \neq 90° + 180°n,\ n \in I$

reflect in x-axis

phase shift right $\frac{\pi}{2}$

6. a) Are the curves $y = \ cosx$ and $y = -\sin(x - \frac{\pi}{2})$ identical?



same graph

b) What are the non-permissible values of x in the equation $cosx\ = -\sin(x - \frac{\pi}{2})$

all values are permissible

<u>Trigonometric Identity</u>: a trigonometric equation that is true for allpermissible values of the variable in the expressions on both sides of the equation.

We can use identities to simplify expressions OR to prove other identities

We can **<u>VERIFY</u>** identities:

- numerically by substituting a value in for the variable. Result: the LHS = RHS
- graphically by graphing the LHS→Y_1 and RHS → Y_2. Result: identical graphs

We can **<u>PROVE</u>** identities by

- algebraically Result: the LHS = RHS for all values of x

1) complex fractions
2) factoring
3) conjugates
4) common denominators

REVIEW: BASIC ALGEBRA INVOLVING RATIONAL EXPRESSIONS

Simplify each of the following:

a) $\cancel{a} \cdot \frac{\cancel{b}}{\cancel{c}} \cdot \frac{\cancel{c}}{\cancel{d}} \cdot \frac{1}{\cancel{b}} = 1$

b) $a \cdot \frac{b^2}{a^2} \cdot \frac{1}{b} = \frac{b}{a}$

c) $\frac{1}{\frac{1}{a}} = a$

d) $\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc}$

$\frac{a}{b} \cdot \frac{d}{c}$

e) $\frac{a+\frac{1}{b}}{b+\frac{a}{b}} = \frac{\frac{ab}{b}+\frac{1}{b}}{\frac{b^2}{b}+\frac{a}{b}} = \frac{\frac{ab+1}{\cancel{b}}}{\frac{b^2+a}{\cancel{b}}}$

$= \frac{ab+1}{\cancel{b}} \cdot \frac{\cancel{b}}{b^2+a}$

$= \frac{ab+1}{b^2+a}$

f) $\frac{a}{b} + \frac{c}{b} = \frac{a+c}{b}$

g) $\frac{a}{b} + \frac{b}{c} = \frac{ac}{bc} + \frac{b^2}{bc}$

$= \frac{ac+b^2}{bc}$

RECIPROCAL IDENTITIES

$csc\theta = \frac{1}{sin\theta}$ $\qquad sec\theta = \frac{1}{cos\theta}$ $\qquad cot\theta = \frac{1}{tan\theta}$

QUOTIENT IDENTITIES

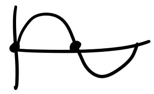
$tan\theta = \frac{sin\theta}{cos\theta}$ $\qquad cot\theta = \frac{cos\theta}{sin\theta}$

VERIFY A POTENTIAL IDENTITY NUMERICALLY & GRAPHICALLY

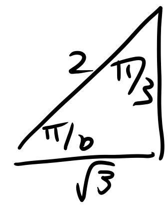
EX. 1: Given $\boxed{cot\theta = \frac{cos\theta}{sin\theta}}$

a) Determine the non-permissible values, in degrees.

$\sin\theta \neq 0$

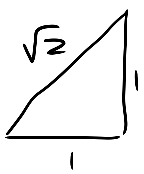


$\theta = 0°, 180°, 360°$ or $180°n,\ n \in I$



b) Numerically verify the following are solutions to the equation

i) $\theta = 45°$ ii) $\theta = \frac{\pi}{6}$



$\cot 45° = \frac{\cos 45°}{\sin 45°}$

$\frac{1}{1} = \frac{\frac{1}{\cancel{\sqrt{2}}}}{\frac{1}{\cancel{\sqrt{2}}}}$

$1 = 1$ ✓

$\cot\left(\frac{\pi}{6}\right) = \frac{\cos(\pi/6)}{\sin(\pi/6)}$

$\frac{\sqrt{3}}{1} = \frac{\frac{\sqrt{3}}{\cancel{2}}}{\frac{1}{\cancel{2}}}$

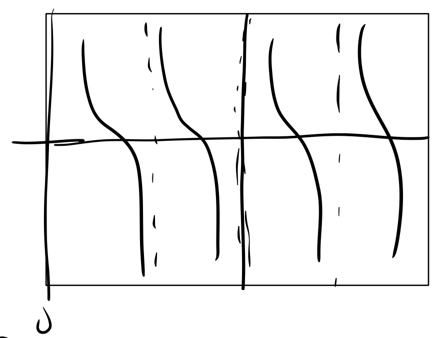
$\frac{\sqrt{3}}{1} = \frac{\sqrt{3}}{1}$ ✓

c) Graphically decide whether the equation could be an identity over the domain $\boxed{-360° \leq x \leq 360°}$

$y_1 = \cot\theta = \frac{1}{\tan\theta}$

$y_2 = \frac{\cos\theta}{\sin\theta}$

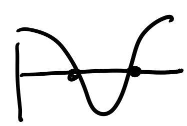
same graph!



USE IDENTITIES TO SIMPLIFY EXPRESSIONS

We can use identities to simplify expressions OR to prove other identities (equations)

EX. 2: Given $\frac{sec\theta}{tan\theta}$



a) Determine the non-permissible values of θ, in radians.

$\sec\theta = \frac{1}{\cos\theta}$ $\quad \tan\theta = \frac{\sin\theta}{\cos\theta}$ $\quad \therefore \cos\theta \neq 0 \Rightarrow \theta \neq 90° + 180°n,\ n \in I$

b) Simplify the expression

$= \frac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}$

$= \frac{1}{\cancel{\cos\theta}} \cdot \frac{\cancel{\cos\theta}}{\sin\theta} = \frac{1}{\sin\theta} = \csc\theta$

EX. 3: Prove the following identities (algebraically)

a) $\sin\vartheta \sec\vartheta \cot\vartheta = 1$

$\cancel{\sin\theta}\left(\frac{1}{\cancel{\cos\theta}}\right)\left(\frac{\cancel{\cos\theta}}{\cancel{\sin\theta}}\right)$

$1 = 1$

b) $\sec\vartheta = \tan\vartheta \csc\vartheta$

$\cancel{\frac{1}{\cos\theta}}$ | $\left(\frac{\cancel{\sin\theta}}{\cos\theta}\right)\left(\frac{1}{\cancel{\sin\theta}}\right)$

$\frac{1}{\cos\theta}$

$\sec\theta = \sec\theta$

c) $\frac{\cot\vartheta}{\csc\vartheta} = \cos\vartheta$

$$\frac{\frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}}$$

$$\frac{\cos\theta}{\cancel{\sin\theta}} \cdot \frac{\cancel{\sin\theta}}{1}$$

$\cos\theta = \cos\theta$

OR

d) $\sec\vartheta(1+\cos\vartheta) = 1 + \sec\vartheta$

$\frac{1}{\cos\theta}(1+\cos\theta)$

$\frac{1}{\cos\theta}+\frac{\cos\theta}{\cos\theta}$

$\sec\theta+1$

$\sec\theta + \cos\theta\sec\theta$

$\sec\theta + \cancel{\cos\theta}\left(\frac{1}{\cancel{\cos\theta}}\right)$

$\sec\theta + 1 \quad = \quad \sec\theta + 1$

OR

$1+\frac{1}{\cos\theta}$

$\frac{\cos\theta}{\cos\theta}+\frac{1}{\cos\theta}$

$\frac{1}{\cos\theta}(\cos\theta+1)$

$\sec\theta(1+\cos\theta)$

e) $\frac{\sin\vartheta+\cos\vartheta}{\sin\vartheta} = 1 + \cot\vartheta$

$\frac{\sin\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$

$1+\cot\theta$

OR

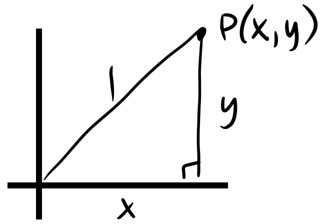
$1+\frac{\cos\theta}{\sin\theta}$

$\frac{\sin\theta}{\sin\theta}+\frac{\cos\theta}{\sin\theta}$

$\frac{\sin\theta+\cos\theta}{\sin\theta}$

PYTHAGOREAN IDENTITY

Other important basic identities are based on the Pythagorean Formula
Use the unit circle diagram and the Pythagorean Formula $a^2 + b^2 = c^2$



- θ is any angle is standard position
- P(x, y) is any point on its terminal arm, at a distance "$r = 1$" from the origin
- Reference triangle is created by drawing a line perpendicular to the x-axis through point P(x, y)

a) sin θ = y	b) cos θ = x	c) tan θ = $\frac{y}{x}$ or $\frac{\sin\theta}{\cos\theta}$

➔ P(x, y) on any circle can be labeled P($\cos\theta$, $\sin\theta$)

➔ Write the relationship between x, y and 1 $\quad x^2 + y^2 = 1$

➔ Write the relationship between $cos\theta$, $sin\theta$, and 1 $\quad \cos^2\theta + \sin^2\theta = 1$

How could you prove the other two Pythagorean identities?

$$\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{1 + \tan^2\theta = \sec^2\theta}$$

$$\boxed{\cot^2\theta + 1 = \csc^2\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\frac{\cos^2\theta}{\sin^2\theta} + \frac{\sin^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

PYTHAGOREAN IDENTITIES

$\boldsymbol{sin^2\theta + cos^2\theta = 1}$ $\qquad$ $\boldsymbol{tan^2\theta + 1 = sec^2\theta}$ * $\qquad$ $\boldsymbol{1 + cot^2\theta = csc^2\theta}$

- Write two different forms of the first Pythagorean identity: **cos^2 x + sin^2 x = 1**

① $\cos^2\theta = 1 - \sin^2\theta$

② $\sin^2\theta = 1 - \cos^2\theta$

③ $-\cos^2\theta = \sin^2\theta - 1$

④ $-\sin^2\theta = \cos^2\theta - 1$

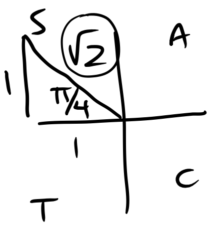
USE THE PYTHAGOREAN IDENTITY

EX. 4: Verify the equation $\tan^2\theta + 1 = \sec^2\theta$

$y_1 = \tan^2 x + 1$

$y_2 = \dfrac{1}{\cos^2 x}$

a) numerically for $\theta = \frac{3\pi}{4}$



$\tan^2\left(\frac{3\pi}{4}\right) + 1 = \sec^2\left(\frac{3\pi}{4}\right)$

$(-1)^2 + 1 = (-\sqrt{2})^2$

$1 + 1 = 2$

$2 = 2$

b) graphically

same graph

EX. 5: Prove the following identities (algebraically)

NOTE: Use any of the basic identities to prove other identities

a) $\mathbf{1 - \cos^2\vartheta = \cos^2\vartheta \tan^2\vartheta}$

$\sin^2\theta \quad | \quad \cancel{\cos^2\theta}\left(\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\right)$

$\sin^2\theta = \sin^2\theta$

b) $\mathbf{\csc\vartheta \cos^2\vartheta + \sin\vartheta = \csc\vartheta}$

$\left(\frac{1}{\sin\theta}\right)\cos^2\theta + \sin\theta$

$\frac{\cos^2\theta}{\sin\theta} + \frac{\sin^2\theta}{\sin\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta}$

$\frac{1}{\sin\theta} = \frac{1}{\sin\theta}$

c) $\cot\vartheta + \tan\vartheta = \csc\vartheta\sec\vartheta$

$$\frac{\cos\theta}{\sin\theta} + \frac{\sin\theta}{\cos\theta} \qquad\Big|\qquad \left(\frac{1}{\sin\theta}\right)\left(\frac{1}{\cos\theta}\right)$$

$$\frac{\cos^2\theta}{\sin\theta\cos\theta} + \frac{\sin^2\theta}{\sin\theta\cos\theta}$$

$$\frac{\cos^2\theta + \sin^2\theta}{\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

d) $(\sec x - 1)(\sec x + 1) = \dfrac{1}{\cot^2 x}$

$$= \sec^2 x - 1 \qquad\Big|\qquad = \frac{1}{\frac{1}{\tan^2 x}}$$

$$= \tan^2 x \qquad\Big|\qquad = 1 \cdot \frac{\tan^2 x}{1}$$

$$\tan^2 x \qquad\Big|\qquad = \tan^2 x$$

e) $\cos^2 x = \dfrac{\cot^2 x}{1 + \cot^2 x}$

$$= \frac{\cot^2 x}{\csc^2 x}$$

$$= \frac{\frac{\cos^2 x}{\sin^2 x}}{\frac{1}{\sin^2 x}} = \frac{\cos^2 x}{\cancel{\sin^2 x}}\left(\frac{\cancel{\sin^2 x}}{1}\right)$$

$$\cos^2 x \qquad\Big|\qquad = \cos^2 x$$

ASSIGNMENT: 1) Worksheet 6.1 Simplify Trigonometric Expressions
2) pg. 296 #1-8, 10, 12, 14-16

PC 12 **WORKSHEET 6.1: SIMPLIFY TRIGONOMETRIC EXPRESSIONS**

- Simplify by expressing each of the following in terms of a single trigonometric function.

a) $\frac{\cos\vartheta}{\sin\vartheta}$ $=\cot\theta$

b) $\frac{\cos^2\vartheta}{\sin^2\vartheta}$ $=\cot^2\theta$

c) $\tan\vartheta\sec\vartheta\cos\vartheta$

$\left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{1}{\cancel{\cos\theta}}\right)\cancel{\cos\theta}$

$\tan\theta$

d) $1-\cos^2\vartheta$

$\sin^2\theta$

e) $\cos^2\vartheta-1$

$-\sin^2\theta$

f) $1+\tan^2\vartheta$

$\sec^2\theta$

g) $\sec^2\vartheta-1$

$\tan^2\theta$

h) $1-\sec^2\vartheta$

$-\tan^2\theta$

i) $1-\csc^2\vartheta$

$-\cot^2\theta$

j) $\underbrace{\sin^2\vartheta+\cos^2\vartheta}+1$

$=1+1$

$=2$

k) $\csc^2\vartheta-\cot^2\vartheta$

$1+\cot^2\theta-\cot^2\theta$

$=1$

l) $\sin^2\vartheta+\cos^2\vartheta+\tan^2\vartheta$

$=1+\tan^2\theta$

$=\sec^2\theta$

m) $\frac{\sqrt{1-\sin^2\vartheta}}{\sqrt{1+\tan^2\vartheta}}$

$=\frac{\sqrt{\cos^2\theta}}{\sqrt{\sec^2\theta}}$

$=\frac{\cos\theta}{\sec\theta}$

$=\cos\theta\cos\theta$

$=\cos^2\theta$

n) $\frac{\sqrt{\sec^2\vartheta-1}}{\sqrt{\csc^2\vartheta-1}}$

$=\frac{\sqrt{\tan^2\theta}}{\sqrt{\cot^2\theta}}$

$=1\left(\frac{\tan\theta}{\cot\theta}\right)$

$=\tan\theta\tan\theta$

$=\tan^2\theta$

o) $\frac{\sin^2\vartheta+\sin\vartheta}{\cos\vartheta+\cos\vartheta\sin\vartheta}$

$\frac{\sin\theta(\cancel{\sin\theta+1})}{\cos\theta(\cancel{1+\sin\theta})}$

$\tan\theta$