PC 12 **SEC 5.4: EQUATIONS AND GRAPHS OF TRIGONOMETRIC FUNCTIONS**

REVIEW: SOLVE TRIGONOMETRIC EQUATIONS

Use processes learned in previous grades to solve equations

- ➔ isolate variables, square roots, factoring (difference of squares, trinomial factoring including decomposition, grouping two and two), quadratic formula, long or synthetic division etc.

Use processes learned in 4.3 notes to find angles given trigonometric ratios

① Ignore sign; Use your calculator or special triangle to find reference angle, θ_r (or points on the unit circle for possible quadrantal angles)

② Use sign of ratio, "ASTC" and θ_r to sketch all possible angles in standard position

③ State the measure(s) of the possible angles in the required domain (use coterminal angles when necessary add/subtract full rotations as needed)

EX. 1: a) Determine the solutions for the trigonometric equation $4sin^2x - 3 = 0$, for the interval $0° \leq x < 360°$

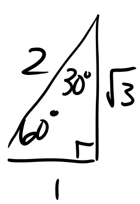
$y = 4\sin(x)^2 - 3$

Method 1: Solve Algebraically

$4\sin^2 x - 3 = 0$

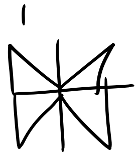
$\sqrt{\sin^2 x} = \sqrt{\frac{3}{4}}$

$\sin x = \pm\frac{\sqrt{3}}{2}$



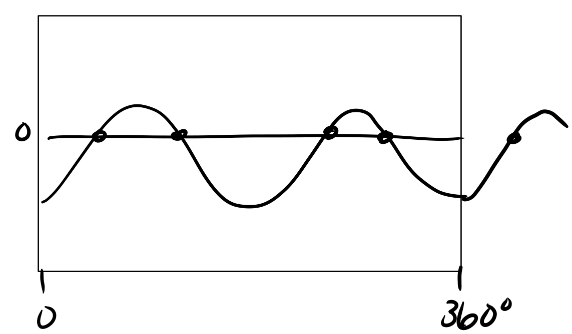
$\theta_R = 60°$

all 4 quad.



$x = 60°\ 120°, 240°\ 300°$

Method 2: Solve Using Graphing Technology



b) Determine the general solution for the trigonometric equation $4sin^2x - 3 = 0$.

$$\left.\begin{aligned} x &= 60° + 180°n \\ &= 120° + 180°n \end{aligned}\right\} n \in I$$

SOLVE TRIGONOMETRIC EQUATIONS (WATCH THE PERIOD)

To solve trigonometric equations with compressed or expanded periods

① Use replacement → let $\theta = "bx"$

② Solve for θ (reference angle (θ_r), quadrants (ASTC), find θ_1 and θ_2)

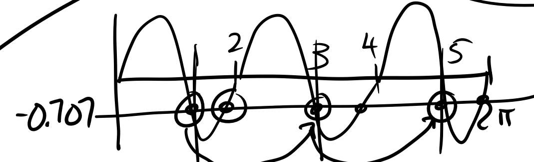
③ Replace each θ with "bx", then solve each equation for x.
→ $bx_1 = \theta_1$ and $bx_2 = \theta_2$

④ To find the general solution
→ add multiples of the compressed or expanded period (p) to each solution (x)
→ $x \pm pn$, $n \in I$

EX. 2: Given the equation $\sin(3x) = -\frac{1}{\sqrt{2}}$

a) How many roots does the equation have for $0 \le x \le 2\pi$? 6

horz comp by $\frac{1}{3}$



$y_1 = \sin 3x$

$y_2 = -\frac{1}{\sqrt{2}}$

$p = \frac{2\pi}{b} = \frac{2\pi}{3}$

b) Solve $\sin(3x) = -\frac{1}{\sqrt{2}}$ for $0 \le x \le 2\pi$. (Give exact solutions)

let $\theta = 3x$

$\sin\theta = \frac{-1}{\sqrt{2}}$

$\theta_R = \pi/4$

$\theta_1 = 5\pi/4$

$\theta_2 = 7\pi/4$

$3x = 5\pi/4$

$x_1 = \boxed{5\pi/12}$

$x_3 = \frac{5\pi}{12} + \frac{2\pi}{3} = \frac{5\pi}{12} + \frac{8\pi}{12} = \boxed{\frac{13\pi}{12}}$

$x_5 = 13\pi/12 + \frac{8\pi}{12} = \boxed{\frac{21\pi}{12} = \frac{7\pi}{4}}$

$3x = 7\pi/4$

$x_2 = \boxed{7\pi/12}$

$x_4 = \frac{7\pi}{12} + \frac{8\pi}{12} = \boxed{\frac{15\pi}{12} = \frac{5\pi}{4}}$

$x_6 = \frac{15\pi}{12} + \frac{8\pi}{12} = \boxed{\frac{23\pi}{12}}$

EX. 3: a) Determine the general solutions for the trigonometric equation $10 = 6cos\left(\frac{\pi}{4}x\right) + 8$

(in radians)

$p = \frac{2\pi}{b} = \frac{2\pi}{\pi/4} = 8$

Method 1: Solve Algebraically

$6\cos\left(\frac{\pi}{4}x\right) + 8 = 10$

$6\cos\left(\frac{\pi}{4}x\right) = 2$

$\cos\left(\frac{\pi}{4}x\right) = \frac{1}{3}$

let $\theta = \frac{\pi}{4}x$

$\cos\theta = \frac{1}{3}$

$\theta_R = \cos^{-1}\left(\frac{1}{3}\right) = 1.23$

$\theta_1 = 1.23$

$\theta_2 = 2\pi - 1.23 = 5.05$

$\frac{\pi}{4}x = 1.23$

$\frac{\pi x}{\pi} = \frac{(1.23)4}{\pi}$

$x = 1.57$

$\pi/4\, x = 5.05$

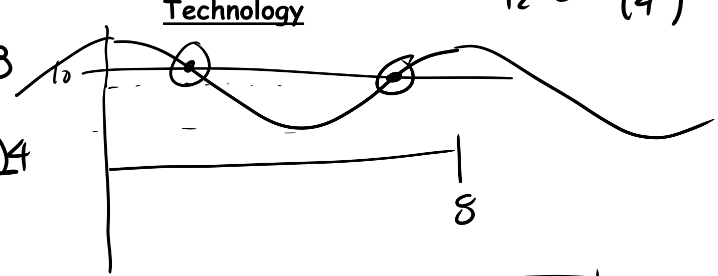
$\frac{\pi x}{\pi} = \frac{5.05(4)}{\pi}$

$x = 6.43$

Method 2: Solve Using Graphing Technology

$y_1 = 10$

$y_2 = 6\cos\left(\frac{\pi}{4}x\right) + 8$



general sol^{n}:

$x = 1.57 + 8n$
$= 6.43 + 8n$ $\}$ $n \in I$

APPLICATIONS OF TRIGONOMETRIC FUNCTIONS

- One of the most useful characteristics of trigonometric functions is their periodicity.
- Mathematicians and scientists use the periodic nature of trigonometric functions to develop mathematical models to predict (interpolate or extrapolate) many natural phenomena
  - ➔ Sunsets, sunrises, and comet appearances… hours of daylight throughout a year
  - ➔ Seasonal temperature changes
  - ➔ Movement of waves in the ocean
  - ➔ Quality of musical sound or vibrations that create sound
  - ➔ Swing of a pendulum
  - ➔ Motion of a piston in an engine
  - ➔ Motion of a ferris wheel
  - ➔ Variations in blood pressure

TIDES AND THEIR SINUSOIDAL MODELS

Tides are the periodic rise and fall of water in the oceans. Therefore, we can use a sinusoidal curve as a model for this periodic motion.

EX. 4: a) Consider the following equation which gives the height , in metres, of the water at time t hours.

$$p = \frac{2\pi}{b} = \frac{2\pi}{\frac{2\pi}{12.4}} = 12.4$$

$$h = 3\cos \overbrace{2\pi \frac{(t-4.5)}{12.4}}^{b} + 5$$

period!

$$3\cos\frac{2\pi}{12.4}(t-4.5)+5$$

b) Determine the amplitude, period, phase shift, and vertical displacement

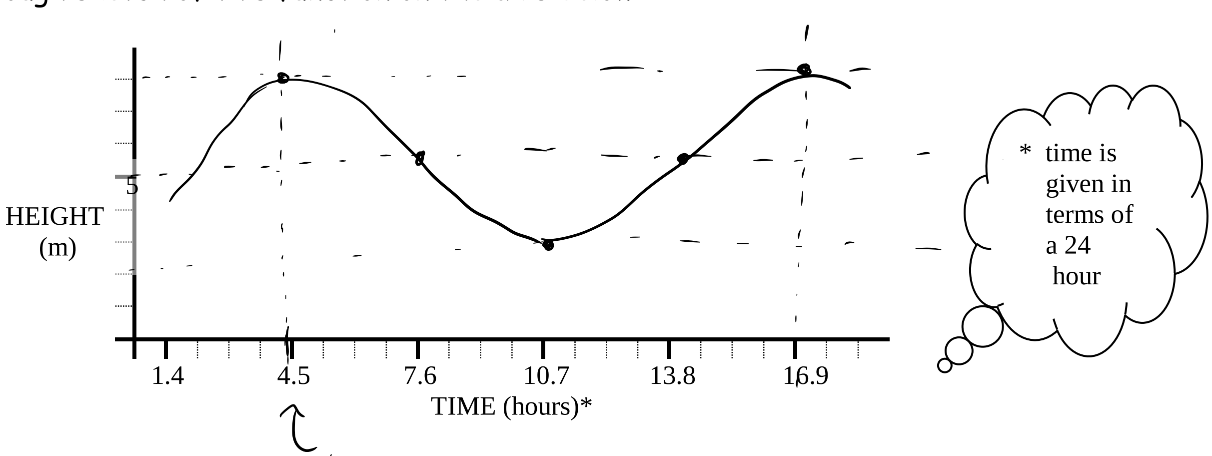
$a = 3$

$p = 12.4$

$ps =$ right 4.5

$vd =$ up 5

c) Draw a rough sketch of this function on the axis below



$4.5 + 12.4$

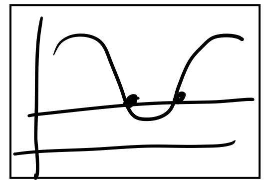
d) i) How were the numbers on the x-axis chosen?

based on the phase shift + period

ii) How were the numbers on the y-axis chosen?

based on the amplitude + vertical displacement

e) Use your graphing calculator to graph then estimate



i) the depth of the water at 2:45 pm (round to the nearest tenth of a metre

$t = 12 + 2.75$ $\boxed{\text{TRACE}}$ $14.75 = 6.39$

$t = 14.75$

depth = 6.4 m

ii) one of the times when the water is 2.5 m deep on the day represented by the equation. (round to the nearest minute)

$y_2 = 2.5$ $x = 9.54$ $0.54 \times 60 = 32$ 9:32 am

DETERMINING THE EQUATION OF A SINUSOIDAL CURVE GIVEN ITS GRAPH

$$\boldsymbol{y = a\sin b(x - c) + d} \text{ or } \boldsymbol{y = a\cos b(x - c) + d}$$

① relocate x- axis

→ **determine amplitude (A) → a**

→ determine vertical displacement (d)

$A = |a| = \frac{maximum\ value - minimum\ value}{2}$

② relocate y-axis

→ identify base graph

→ determine the phase shift (c)

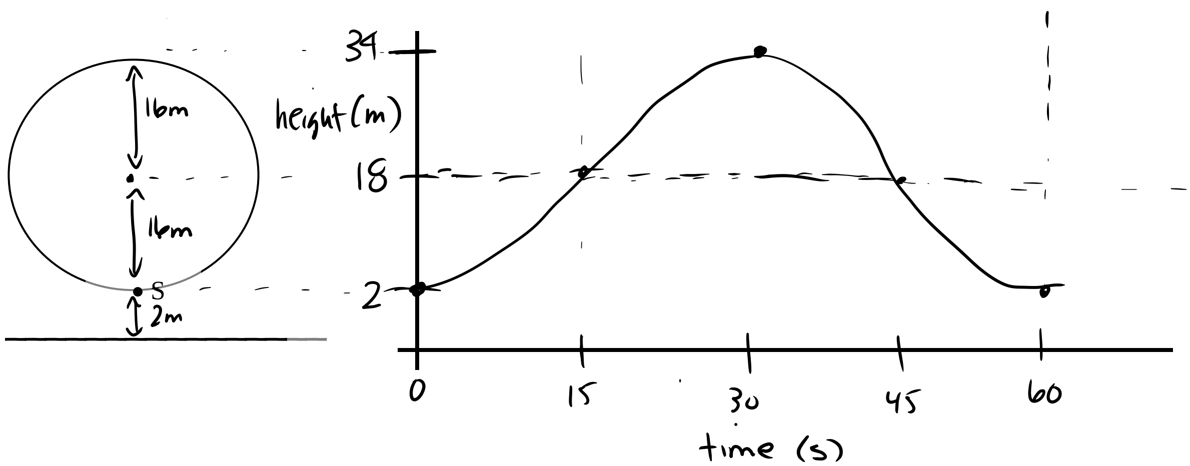
→ **determine period (p) → b**

$p = \frac{2\pi}{b}$ *then* b = $\frac{2\pi}{P}$

FERRIS WHEELS AND THEIR SINUSOIDAL MODELS

EX. 5: A ferris wheel has a radius of 16 m. It rotates once every 60 s. Passengers get on at its lowest point, which is 2 m above ground level. Suppose you get on at its lowest point (S) and the wheel starts to rotate

a) Graph how your height above the ground varies during the first cycle.



b) Write the equation that expresses your height as a function of the elapsed time.

$a = 16$

$b = \frac{2\pi}{60}$

(ps) $c = 0$ ← reflected cos curve

(vd) $d = 18$

$$h = -16\cos\frac{2\pi}{60}t + 18$$

c) Estimate your height above the ground after 45 s.

t

$$h = -16\cos\frac{2\pi(45)}{60} + 18 = 18 \text{ m}$$

$\frac{3\pi}{2}$

d) Estimate the first time when your height is 29 m above the ground

h

$$29 = -16\cos\frac{2\pi t}{60} + 18$$

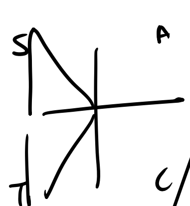
$-18 \qquad -18$

$$11 = -16\cos\frac{2\pi t}{60}$$

$$\frac{-11}{16} = \cos\frac{2\pi t}{60}$$

let $z = \frac{2\pi t}{60}$ $\qquad \cos z = \frac{-11}{16}$

$$\theta_R = \cos^{-1}\left(\frac{11}{16}\right) = 0.81$$



$z_1 = \pi - 0.81 = 2.3$

$z_2 = \pi + 0.81 = 3.95$

$\frac{2\pi t}{60} = 2.3$

$\frac{2\pi t}{2\pi} = \frac{139.8}{2\pi}$

$\boxed{t = 22.24 \text{ s}}$

$\frac{2\pi t}{60} = 3.95$

$\frac{2\pi t}{2\pi} = \frac{237}{2\pi}$

$t = 37.72 \text{ s}$

MODEL ELECTRIC POWER

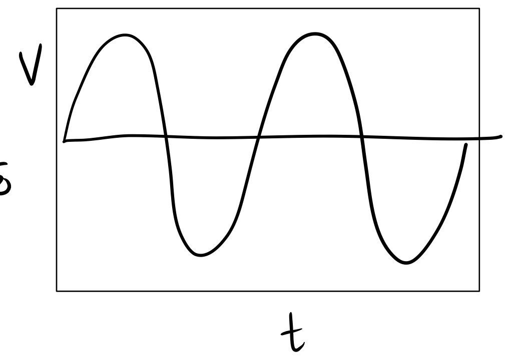
The electricity coming from power plants into your house is alternating current (AC). This means that the direction of current flowing in a circuit is constantly switching back and forth. In Canada the current makes 60 complete cycles each second. The voltage can be modeled as a function of time using sine function $V = 170 sin 120\pi t$

EX. 6: In some Caribbean countries, the current makes 50 complete cycles each second and the voltage is modeled by $V = 170 sin 100\pi t$

a) What is the period of the function in these countries?

$P = \frac{2\pi}{100\pi} = \frac{1}{50}$ OR $P = 1\text{cycle}$

$\frac{1\text{sec}}{50\text{cycles}} = \frac{1}{50}$



b) Graph the voltage function over 2 cycles

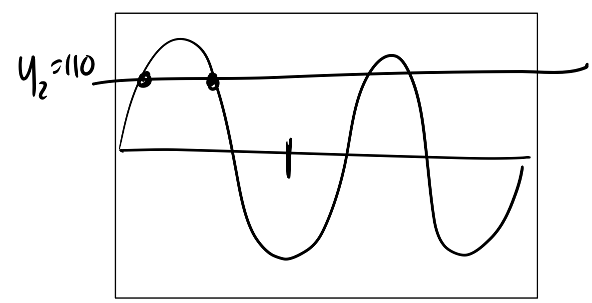
$2\text{ cycles} = 2\left(\frac{1}{50}\right) = 0.04 \leftarrow X_{max}$

c) How many times does the voltage reach 110V in the first second?

2 times in 0.02 sec

$\downarrow \times 50$ $\quad\quad$ $\downarrow \times 50$

100 x $\quad\quad$ 1 sec



MODEL HOURS OF DAYLIGHT

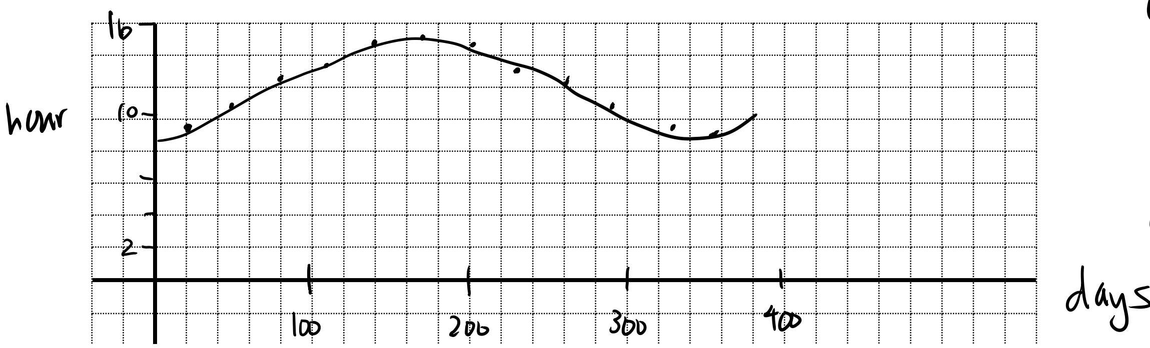
EX. 7: Windsor, Ontario, is located at latitude 42° N. The table shows the number of hours of daylight on the 21st day of each month as the day of the year on which it occurs for this city

Hours of Daylight by Day of the Year for Windsor, Ontario

Jan 21   Feb 21   July 21   Dec 21

21	52	80	111	141	172	202	233	264	294	325	355
9.62	10.87	12.20	13.64	14.79	15.28	14.81	13.64	12.22	10.82	9.59	9.08

a) Draw a scatterplot for the number of hours of daylight, h, in Windsor, Ontario on the day of the year, t.



$a = \frac{M-m}{2} = \frac{15.28-9.08}{2} = 3.1$

$b = \frac{2\pi}{365}$

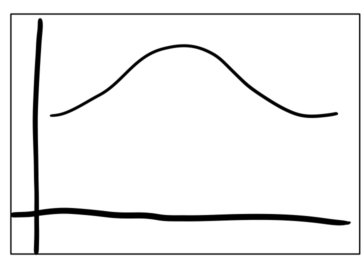
$c = 172$

$d = \frac{M+m}{2} = \frac{15.28+9.08}{2} = 12.18$

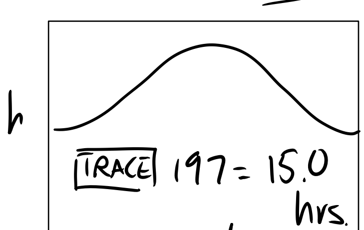
b) Write the sinusoidal function that models the number of hours of daylight.

$$h = 3.1\cos\frac{2\pi}{365}(d-172) + 12.18$$

c) Graph the function you obtained from (b) on your graphing calculator



d) Estimate the number of hours of daylight on July 16th



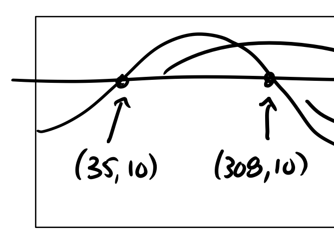
J - 31
F - 28
M - 31
A - 30
M - 31
J - 30
Ju - 16

Day 197

d

e) When are there approximately ten hours of daylight

$y_2 = 10$



Feb 4

Nov 4

ASSIGNMENT: 1) W.S. 5.4: APPLICATIONS OF TRIG FUNCTIONS

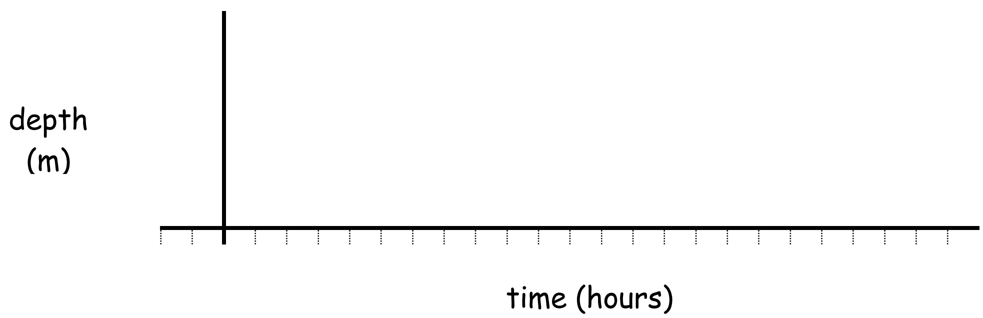
2) pg.275 #1-7,10,12,14,16*,19*,23*

PC 12 WORKSHEET 5.4: APPLICATIONS OF TRIG FUNCTIONS

A. TIDES AND THEIR SINUSOIDAL MODELS

1. You are the captain of an oil tanker that needs to dock at the port of Traversville. Your navigator went crazy and jumped overboard 500 miles out in the Atlantic Ocean, so it is your responsibility to get the ship in and out of port safely. Your charts indicate that the time between high and low tide at this port is 6.2 hours and the average depth of the water in the port is 40 metres; at high tide, the depth is 50 metres.

   a) Sketch a graph of the depth of the water in the port over time if the relationship between time and depth is sinusoidal and the chart indicates that on this date there is a high tide at 12:00 noon.



   b) By letting "t" represent the number of hours since 12:00 noon, write an equation

   c) If the boat requires a depth of 35 metres or more of water to avoid being run aground, assuming that you do not unload any oil:

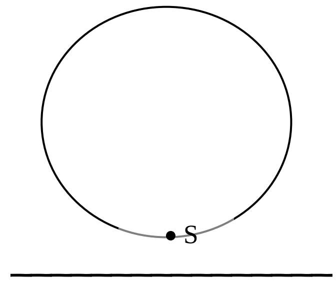
      i) How many minutes before noon can you enter port?

      ii) How many minutes after noon must you leave port?

      iii) Between what times are you able to be in port?

B. FERRIS WHEELS AND THEIR SINUSOIDAL MODELS

2. A ferris wheel has a radius of 20 m. It rotates once every 40 s. Passengers get on at its lowest point, which is 1 m above ground level. Suppose you get on at its lowest point (S) and the wheel starts to rotate



a) Graph how your height above the ground varies during the first cycle.

b) Write the equation that expresses your height as a function of the elapsed time.

c) Estimate your height above the ground after 45 s.

d) Estimate the first time when your height is 35 m above the ground.

C. MORE…APPLICATIONS WITH SINUSOIDAL MODELS

3. According to the famous news weather network W.E.T - TV, the day with the largest amount of daylight is June 21^{st} (the 172^{nd} day of the year) and the day with the shortest amount of daylight is coming up on December 21^{st} (the 355^{th} day of the year). There are 15 hours of daylight on June 21 and 9 hours of daylight on December 21^{st} here on Vancouver Island.

    a) Sketch a graph of the hours of daylight over time if the relationship between time and hours is sinusoidal

    b) The amount of daylight follows a sinusoidal pattern where "t" is the number of days into the year and L is the number of hours of daylight. Write an equation.

    c) When are there approximately ten hours of daylight?

    d) Approximately how many hours and minutes of daylight should there be today?