PC 12 **SEC 4.4: INTRO TO TRIGONOMETRIC EQUATIONS**

INVESTIGATE: SOLVING QUADRATIC AND TRIGONOMETRIC EQUATIONS

Solve the quadratic equation for x	Solve the trigonometric equation for $\boxed{\sin\theta}$
$\sqrt{2}x + 1 = 0$ (−1 −1) $\frac{\cancel{\sqrt{2}}x}{\cancel{\sqrt{2}}} = \frac{-1}{\sqrt{2}}$ $\quad x = -\frac{1}{\sqrt{2}}$	$\sqrt{2}\sin\theta + 1 = 0$ (−1 −1) $\frac{\cancel{\sqrt{2}}\sin\theta}{\cancel{\sqrt{2}}} = \frac{-1}{\sqrt{2}}$ $\quad \sin\theta = \frac{-1}{\sqrt{2}}$
$3x + 1 = 0$ $3x = -1$ $x = -\frac{1}{3}$	$3\sin\theta + 1 = 0$ $3\sin\theta = -1$ $\sin\theta = -\frac{1}{3}$
$x = \sqrt{3} - x$ $2x = \sqrt{3}$ $x = \frac{\sqrt{3}}{2}$	$\sin\theta = \sqrt{3} - \sin\theta$ $2\sin\theta = \sqrt{3}$ $\sin\theta = \frac{\sqrt{3}}{2}$
$4x^2 = 1$ $\sqrt{x^2} = \sqrt{\frac{1}{4}}$ $\quad x = \pm\frac{1}{2}$	$4\sin^2\theta = 1$ $\sqrt{\sin^2\theta} = \sqrt{\frac{1}{4}}$ $\quad \sin\theta = \pm\frac{1}{2}$
$x^2 - x = 0$ $x(x-1) = 0$ $x = 0, 1$	$\sin^2\theta - \sin\theta = 0$ $\sin\theta(\sin\theta - 1) = 0$ $\sin\theta = 0, 1$
$x^2 + 2x + 1 = 0$ $\quad 1 + 1 = 2$ $(x+1)(x+1) = 0$ $\quad 1 \times 1 = 1$ $x = -1$	$\sin^2\theta + 2\sin\theta + 1 = 0$ $(\sin\theta + 1)(\sin\theta + 1) = 0$ $\sin\theta = -1$
$2x^2 - 5x - 3 = 0$ $\quad -6 + 1 = -5$ $2x^2 - 6x + 1x - 3 = 0$ $\quad -6 \times 1 = -6$ $2x(x-3) + 1(x-3) = 0$ $(2x+1)(x-3) = 0$ $x = -\frac{1}{2}, 3$	$2\sin^2\theta - 5\sin\theta - 3 = 0$ $\quad$ Note: $\sin^2\theta = (\sin\theta)^2$ $2\sin^2\theta - 6\sin\theta + 1\sin\theta - 3 = 0$ $2\sin\theta(\sin\theta - 3) + 1(\sin\theta - 3) = 0$ $(2\sin\theta + 1)(\sin\theta - 3)$ $\boxed{\sin\theta = -\frac{1}{2}}, 3$
$x^2 + x - 1 = 0$ $x = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$ $= \frac{-1 \pm \sqrt{5}}{2}$	$\sin^2\theta + \sin\theta - 1 = 0$ $\sin\theta = \frac{-1 \pm \sqrt{(1)^2 - 4(1)(-1)}}{2(1)}$ $\quad \sin\theta = \frac{-1 \pm \sqrt{5}}{2}$

NOTE: When you solve a trigonometric equation you actually go further and solve for θ

SOLVE TRIGONOMETRIC EQUATIONS

Use processes learned in previous grades to solve equations

➔ isolate variables, square roots, factoring (difference of squares, trinomial factoring including decomposition, grouping two and two), quadratic formula, long or synthetic division etc.

Use processes learned in 4.3 notes to find angles given trigonometric ratios

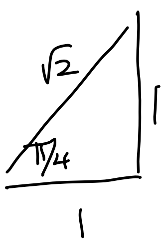
① Ignore sign; Use your calculator or special triangle to find reference angle, θ_r (or points on the unit circle for possible quadrantal angles)

② Use sign of ratio, "ASTC" and θ_r to sketch all possible angles in standard position

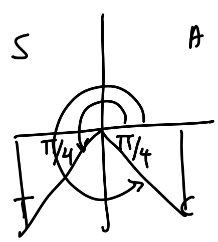
③ State the measure(s) of the possible angles in the required domain (use coterminal angles when necessary add/subtract full rotations as needed)

ex. Solve $\sqrt{2}\sin\theta + 1 = 0$ for $0 \le \theta \le 2\pi$ (Give exact answers)

$\sin\theta = \frac{-1}{\sqrt{2}}$



$\theta_R = \frac{\pi}{4}$



$\theta_1 = \frac{5\pi}{4}$

$\theta_2 = \frac{7\pi}{4}$

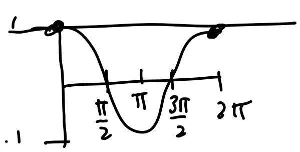
EX. 1: Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1, \; \cancel{2\pi}\ 0 \le \theta \le 2\pi$

$-\cos\theta \quad +1 \qquad -\cos\theta \quad +1$

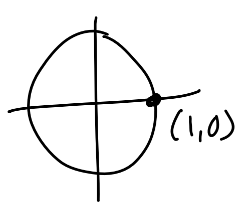
$2\cos\theta = 2$

$\cos\theta = 1$



$\theta_1 = 0$

$\theta_2 = 2\pi$

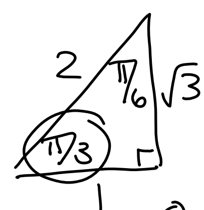


b) $4\sec x + 8 = 0, \; 0° \le x < 360°$

$\sec x = \frac{-8}{4}$

$\sec x = -2$

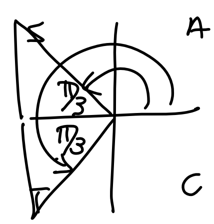
$\cos x = \frac{-1}{2}$



$\theta_R = \pi/3$

$\theta_1 = \frac{2\pi}{3}$

$\theta_2 = \frac{4\pi}{3}$



FACTOR TO SOLVE A TRIGONOMETRIC EQUATION

EX. 2: Solve each trigonometric equation in the specified domain.

a) $\boldsymbol{tan^2\theta - 5tan\theta + 4 = 0},\ 0 \le \theta < 2\pi$

let $a = \tan\theta$

$a^2 - 5a + 4 = 0$ $\quad$ $-4 + -1 = -5$, $\quad -4 \times -1 = 4$

$(a-4)(a-1) = 0$

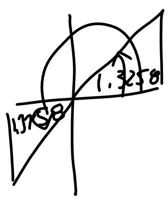
$a - 4 = 0$ or $a - 1 = 0$

$a = 4$ $\qquad a = 1$

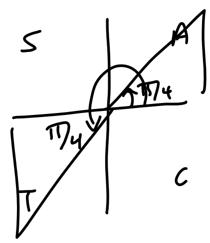
$\tan\theta = 4$ $\qquad \tan\theta = 1$

$\theta_R = \tan^{-1}(4)$

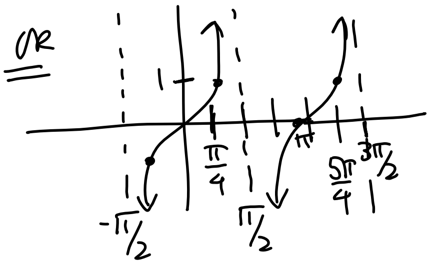
$= 1.3258$



$\theta_R = \frac{\pi}{4}$



OR



$$\boxed{\begin{aligned}\theta_1 &= \frac{\pi}{4}\\ \theta_2 &= \frac{5\pi}{4}\\ \theta_3 &= 1.3258\\ \theta_4 &= \pi + 1.3258\\ &= 4.4674\end{aligned}}$$

b) $\boldsymbol{cos^2\theta - cos\theta - 2 = 0},\ 0° \le x < 360°$

let $a = \cos\theta$

$a^2 - a - 2 = 0$ $\quad$ $-2 + 1 = -1$, $\quad -2 \times 1 = -2$

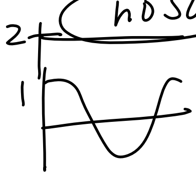
$(a-2)(a+1) = 0$

$a - 2 = 0$ $\qquad a + 1 = 0$

$a = 2$ $\qquad a = -1$

$\cos\theta = 2$ $\qquad \cos\theta = -1$

no soln



$\theta = 180°$

$$\boxed{\theta_1 = 180°}$$

GENERAL SOLUTION TO A TRIGONOMETRIC EQUATION

Use Coterminal Angles in General Form: to write an infinite number of angles are coterminal with any given angle

- found by adding or subtracting multiples of one full rotation,

In DEGREES: $\theta + 360^\circ n$, where n is any integer ($n \varepsilon I$)

In RADIANS: $\theta + 2\pi n$, where n is any integer ($n \varepsilon I$)

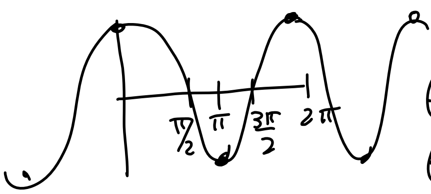
} * for sine and cosine

for tan use $180^\circ n$ or πn

EX. 3: **a)** If $\boxed{\cos^2\theta - 1 = 0}$, solve for θ in the domain $0° \leq x < 360°$. Give exact solutions

METHOD 1: Use Square Roots

$$\cos^2\theta - 1 = 0$$
$$\sqrt{\cos^2\theta} = \sqrt{1}$$
$$\cos\theta = \pm 1$$



$$\theta_1 = 0^\circ$$
$$\theta_2 = 180^\circ$$

METHOD 2: Using Factoring

$$\cos^2\theta - 1 = 0$$
$$(\cos\theta + 1)(\cos\theta - 1) = 0$$
$$\cos\theta + 1 = 0 \quad \text{or} \quad \cos\theta - 1 = 0$$
$$\cos\theta = -1 \qquad \cos\theta = 1$$
$$\theta_1 = 0$$
$$\theta_2 = 180^\circ$$

b) Determine the general solution for $\cos^2\theta - 1 = 0$, where the domain is real numbers measured in degrees.

$$\left.\begin{aligned}\theta &= 0^\circ + 360^\circ n \\ &= 180^\circ + 360^\circ n\end{aligned}\right\} n \in I$$

or $\theta = 0 + 180^\circ n,\ n \in I$

or $\theta = 180^\circ n,\ n \in I$

How can you show algebraically that $\boxed{(2n+1)\left(\frac{\pi}{2}\right)}, n \epsilon I$ and $\boxed{\frac{\pi}{2} + \pi n,}\ n \epsilon I$ are equivalent

$$(2n+1)\left(\frac{\pi}{2}\right) = \frac{\pi}{2} + \pi n$$
$$\frac{\cancel{2}n\pi}{\cancel{2}} + \frac{\pi}{2}$$
$$n\pi + \frac{\pi}{2}$$
$$\pi/2 + \pi n = \pi/2 + \pi n$$

ASSIGNMENT:	**1) Worksheet 4.4: Solve Trigonometric Equations**
	2) pg. 211 # 1-5, 7, 8, 10, 11, 13, 16, 18

PC 12 **WORKSHEET 4.4: TRIGONOMETRIC RATIOS**

- **Solve each trigonometric equation for θ. Determine the general solution for each.**
  Note: These are the equations from the investigate on p. 22 in notes

a) $\sqrt{2}\sin\theta + 1 = 0$

b) $3\sin\theta + 1 = 0$

c) $sin\theta = \sqrt{3} - \sin\theta$

d) $4sin^2\theta = 1$

e) $sin^2\theta - sin\theta = 0$

f) $sin^2\theta + 2sin\theta + 1 = 0$

g) $2sin^2\theta - 5sin\theta - 3 = 0$

h) $sin^2\theta + sin\theta - 1 = 0$